Quantitative Easing and Fiscal Policy Effectiveness

Antzelos Kyriazis*

Yale University†

November 2023

Abstract

This paper studies the effects of fiscal policy on aggregate economic activity and inequality when the monetary authority follows conventional and unconventional policies. First, I build a three-agent Preferred Habitat New Keynesian (PHANK) model with a banking sector in which QE matters for the determination of output in the short run. I analytically derive the fiscal multiplier and show that it decreases in the presence of countercyclical QE policies, even at the zero lower bound. A calibration of the model for the US economy yields fiscal and QE multipliers close to 3 when the monetary authority pegs the short-term policy rate. The optimal fiscal and QE policies are expansionary at the ZLB. Second, I also consider a medium-scale HANK model to further study the distributional effects of fiscal expansions and recompute the fiscal multipliers under active fiscal policy, passive monetary policy and QE. In the enhanced model, the government spending multiplier at the ZLB is 1.041. Countercyclical QE after a fiscal expansion reduces consumption inequality in the medium run but increases wealth inequality. In the short-run those effects are reversed.

---

*Current affiliation: Templeton Global Macro, Franklin Templeton. Email: antzelos.kyriazis@franklintempleton.com
Any views expressed herein are the views of the author and do not represent the views of the organization he works for.

†This is a slightly revised version of Chapter 1 of my PhD dissertation, with the title "Quantitative Easing and Fiscal Policy Effectiveness". The original paper was written while I was a graduate student at Yale University.

1 Introduction

A central issue in macroeconomics is the effectiveness of fiscal policy. Given the distributional role attributed to fiscal policy, effectiveness does not refer only to stimulating economic activity but also to reducing inequality. This paper revisits this topic and studies the effectiveness of fiscal policies in the presence of conventional and unconventional monetary policies, price rigidities, and heterogeneity across households, both in regular times and in crises. The intention is to highlight the effects of unconventional monetary policies on the aggregate and distributional effects of fiscal expansions.

The aggregate effects of fiscal expansions are related to what is known in macroeconomics as the fiscal multiplier. The logic behind this notion is that a positive fiscal shock increases aggregate demand in the short run. Firms satisfy this demand by increasing their labor demand, which implies higher employment, production, and wages. Nevertheless, higher wage income boosts private consumption of at least some types of households, raising aggregate demand even more and triggering another round of positive movements in employment, income, and production. Of course, behind the reaction of the firms is the assumption of sticky prices. If prices were not sticky, the price level would quickly jump in reaction to higher aggregate demand and offset these positive effects on economic activity.

On the other hand, behind the reaction of the households is the assumption that some households are willing to increase their consumption significantly after a positive shock and do not necessarily increase savings to protect themselves against higher taxes in the future. In addition, another critical factor affects the magnitude of the multiplier, which is the reaction of the monetary authority. The multiplier is larger if monetary policy is accommodative and pegs the nominal interest rate since short-term savers see the expected real interest rate falling when prices increase and, as a result, consume more today. However, central banks have recently adopted quantitative easing policies on top of traditional measures, raising the question of how much more or less effective fiscal policy will be when QE is also taking place.

The second issue is related to the other main goal of the fiscal authorities, redistribution. In particular, after a positive fiscal shock, liquidity-constrained households with high marginal propensities to consume become better off since their labor income increases. The unconstrained households with high levels of financial wealth gain less from an increase in real wages. However, their financial income and savings decisions are affected by the movements in asset prices induced by the fiscal shock. Usually, after a fiscal expansion that increases government debt, the price of debt falls. However, asset prices also depend on the reaction of monetary policy. By influencing asset prices, monetary policy affects the returns earned by saving households on previous savings, leading to changes in their savings decisions. These changes can make the wealth inequality gap higher over time. Hence, fiscal authorities need to be aware of the distributional effects caused by the choices of central banks when designing fiscal programs that also target inequality.

This paper aims to provide insights into the two issues raised above. In the first part of the analysis, I consider an analytically tractable three-agent Preferred Habitat New Keynesian (PHANK) model with QE policies. In this model, there are three types of households: short-term savers, who save only by holding short-term bank deposits; long-term savers, who hold long-term government debt and own the firms and the banks; and hand-to-mouth consumers, who do not save but consume their labor income. In this way, I introduce heterogeneity in *MPCs*, which makes the fiscal multiplier high, and examine the effects of fiscal and QE policies on different types of households. In addition, in the model, private banks are operating under financial frictions needed to make QE policies effective, and goods-producing firms set their prices in a sticky fashion. The central bank, and the government constitute the policy sector. I derive a dynamic IS curve that summarizes the optimality conditions of all three types of households and private banks. In addition to the short-term policy rate, the IS curve also depends on fiscal policy and the quantitative easing decisions of the central bank, implying that QE matters in the short run for the determination of output. Then I use the model to derive the fiscal multiplier analytically.

I calibrate the model to the US economy using parameter values comparable to those used in previous studies on the fiscal multiplier. I find a fiscal multiplier on impact equal to 0.638 away from the ZLB. I model the central bank's asset purchases as a countercyclical Taylor rule, and this makes the multiplier in the short-run depend negatively on the responsiveness of QE to economic activity. The reason is that the countercyclical response of asset purchases leads to a reduction in the price of government debt, making the bonds a more attractive investment for long-term savers, resulting in higher savings and lower aggregate consumption. The last result is true when the ZLB is binding. At the ZLB, the fiscal multiplier is around 1.93 and is close to 3 when the central bank is not implementing countercyclical QE. The model also shows that a higher target for the central bank's long-term bond purchases reduces the multiplier. I repeat this exercise for various models nested within the three-agent model and reach similar qualitative conclusions. I also derive the quantitative easing multiplier by altering the assumption on how QE is determined, making this a purely discretionary choice. The QE multiplier is 0.426 in normal times and exceeds 3 at the ZLB.

Regarding the effects of policy on the different types of households, an increase in government spending tends to increase the consumption of each type of household due to higher labor income. The long-term savers also experience adverse effects on their consumption from holding more bonds after the fiscal shock when the central bank reacts countercyclically and sells bonds. However, this countercyclical response of the central bank leads to a minor decrease in real wages and marginal costs, resulting in higher profits after the fiscal shock. Short-term savers consume less because they have to pay higher taxes after the fiscal shock.

On balance, the countercyclical QE response of the central bank weakens the decrease in consumption inequality between short-term savers and constrained households caused by fiscal policy. This results from the lower nominal and real interest rates, which lead short-term savers to consume more. In addition, although countercyclical QE tends to lower the effects of fiscal policy on aggregate demand and the real wage, and as a result, on the consumption of the constrained households, it lowers the consumption of the long-term savers even more due to their increased bond holdings. Thus, constrained households' consumption rises more when compared to long-term savers. Short-term savers also gain relative to long-term savers from countercyclical QE because of the lower real interest rate, which incentivizes them to consume more.

The model also assesses the optimal fiscal and quantitative easing policies at the zero lower bound. I assume a utilitarian government and derive a new social welfare function that includes the consumption and labor supply levels of all types of households in the economy weighted by their corresponding sizes in the population. The problem of optimal fiscal policy at the zero lower bound is analyzed first, assuming that a Taylor-type rule determines QE. I find that the optimal deviation of government spending from its steady state value is positive and equal to 5.94%. I also provide a comparative statics analysis for the optimal choice with respect to other parameters of interest. Then the government is allowed to pick both government spending and the central bank's asset holdings at the ZLB. In this case, it is optimal for asset holdings to be positive when combined with a positive choice for government spending. That is, QE should increase together with government spending at the ZLB, implying that the countercyclical Taylor rule, which would dictate reduced asset purchases after the increase in government spending, is not optimal. The optimal QE deviation from the steady state is 1.39%, and the optimal value for government spending deviation is 5.44%. Since asset purchases are positive and QE stimulates the economy, less fiscal action is needed, so lower taxes need to be imposed on households to cover the fiscal expansion, creating fewer adverse welfare effects.

I consider a medium-scale heterogeneous agent New Keynesian model in the second part of the analysis. This model preserves the qualitative properties of the three-agent model discussed above and can better address the inequality issues by construction. After calibrating the HANK model to the US economy for the period 2008-2021, I find that at the zero lower bound, with countercyclical QE policies and active fiscal policy, the government spending multiplier on impact is 1.041. The stimulating role of quantitative easing is weaker in this model relative to the three-agent model since, in the absence of QE, the multiplier becomes 1.061.

In terms of redistribution, the fiscal shock reduces inequality in the enhanced model. Specifically, consumption inequality falls because the higher labor income allows the constrained households to increase their consumption while richer households reduce their consumption over time due to a decrease in profits. Countercyclical QE again tends to reduce consumption inequality over the medium run since profits fall more in this case. However, wealth inequality increases since labor income is lower and poorer households are not able to accumulate illiquid assets as the richer households do over time, because of the high adjustment costs related to this asset. Also, inflation is higher under countercyclical QE, since higher interest rates in the future provide wealth effects to debt holders which are not cancelled out by fiscal policy, since fiscal policy is active, and higher inflation reduces the liquid return so poorer households hold less liquid assets.

Away from the ZLB, and assuming that the central bank follows a countercyclical Taylor rule for the short-term nominal interest rate and does not purchase assets, the fiscal multiplier is lower and equal to 0.791. The main reasons for the reduction are two. First, the strong response of lump-sum taxes, which reduces the available income of households and creates expectations of high taxation in the future, and second, the central bank's strong countercyclical interest rate response to the increase in output and inflation.

The main contribution of this paper is to provide the analytics for the fiscal and the quantitative easing multipliers in a preferred-habitat New Keynesian model with central bank asset purchases and the corresponding comparative statics. The paper also discusses the distributional effects of fiscal expansions and how those are affected by QE. It also provides analytical solutions for the optimal fiscal and quantitative easing policies at the ZLB after deriving a new social welfare function. In addition, the paper analyzes the size of the fiscal and the quantitative easing multipliers in a medium-size HANK model with QE, and examines how QE affects the fiscal multiplier and the fiscal effects on inequality. To my knowledge, this is the first paper that studies these questions by combining heterogeneous agents, the fiscal theory of the price level, financial frictions, price and wage rigidities, and QE.

Related Literature: This paper is related to many different parts of the literature. First, it is related to papers that derive analytically and characterize the fiscal multiplier. Such papers include Woodford (2011), Christiano et al. (2011), and Cochrane (2017). In this paper, I provide an analytical framework with different types of agents and QE policies and a fully specified medium-scale HANK model to analyze the fiscal multiplier and provide insights about distribution. Farhi and Werning (2016) study and derive the fiscal multipliers in various frameworks, including models of closed and open economies and complete and incomplete markets. However, their models do not incorporate unconventional monetary policies. Another recent model that incorporates heterogeneity and allows for redistribution across agents in an analytical framework is Bilbiie (2018), but there are no banks and unconventional policies in that paper.

This study also relates to papers such as Dupor et al. (2018) and Hagedorn et al. (2019b), which characterize fiscal policy using medium-scale HANK models for different regions or a single economy, respectively. The HANK model I present is different because it includes a private banking sector and QE policies on the monetary authority's side, which create extra distributional effects and change the transmission mechanism of the fiscal shock and the multiplier.

This paper is related to papers using New Keynesian models with a banking sector, and financial frictions to explore the effects of quantitative easing policies on the economy, such as Gertler and Karadi (2011), Gertler and Karadi (2013), and Sims et al. (2021). The main difference in this paper is that I use this framework to measure and characterize the fiscal multiplier and see how conventional and unconventional monetary policies interact with fiscal policy. Moreover, the models I employ here can better deal with the issue of heterogeneity and inequality because the previous papers focus either on a representative agent or an economy with two unconstrained types of households. Another line of models which incorporate QE and fiscal policy are big-scale policy models such as Dimakopoulou et al. (2022). I depart from these models by studying a simplified theoretical framework which allows to analyze the mechanisms at play.

Finally, this study is related to the more general and growing HANK literature that combines incomplete market models with models that include nominal rigidities. Some indicative papers are Ravn and Sterk (2017), Kaplan et al. (2018), Auclert and Rognlie (2018), Auclert et al. (2018), Hagedorn et al. (2019a), Auclert et al. (2020). However, these papers do not incorporate unconventional monetary policy. Other HANK models incorporating QE policies are Cui and Sterk (2021), Lee (2020), Kyriazis (2022), and Sims et al. (2022) but these papers do not focus on the fiscal multiplier.

Paper Organization: The rest of the paper is organized in the following way. Section 2 contains a tractable Three-Agent New Keynesian model that analytically shows how QE policies affect the fiscal multiplier in normal times and in a liquidity trap, and how the distributional effects of fiscal expansions are affected by QE. Later in this section, the problem of choosing optimally fiscal and quantitative easing policies at the ZLB is discussed. Section 3 contains the medium-scale HANK model and the quantitative analysis.

2 A Tractable PHANK Model

In this section, I present an analytically tractable three-agent Preferred-Habitat New Keynesian (PHANK) model that analyzes the effects of unconventional monetary policy on the fiscal multiplier. The model extends the work of Sims et al. (2021). There are three types of households: short-term savers, who save only by holding short-term bank deposits; long-term savers, who hold long-term government debt and own the firms and the banks; and hand-to-mouth consumers, who do not save but consume their labor income. These three types of households try to match the three main categories of households in the real world: the households who mostly save by holding simple bank accounts, the wealthier households who hold equity and own the firms, and the poorer households who depend only on their labor income and do not have access to savings vehicles. In addition, the model includes private banks, goods-producing firms, the central bank, and the government.

I first discuss the optimization problems and the constraints faced by each agent. Then I derive the log-linearized aggregate equilibrium conditions and study the fiscal and the QE multipliers both outside and at the zero lower bound. I also discuss the distributional effects of fiscal shocks under various assumptions regarding monetary policy. Then the section is closed with questions related to optimal fiscal and QE policies.

2.1 Long-term Savers

Long-term savers, or parents, constitute the first category of households. The size of the parent cohort is $\eta_p \in (0,1)$. These households derive utility from consuming C_{pt} and disutility from working L_{pt}. They are the owners of the intermediate goods firms and the financial intermediaries. They save by holding only long-term government debt $q_t B_{pt}$, where q_t is the price of the debt, and receive a transfer from the government. In every period, they choose consumption, labor supply, and government bonds to maximize lifetime utility

$$\max_{\{C_{pt}, L_{pt}, B_{pt}\}_{t=0}^{\infty}} U_p = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_p^t \left(\frac{C_{pt}^{1-\sigma} - 1}{1-\sigma} - \mu_L \frac{L_{pt}^{1+\nu}}{1+\nu} + \mu_G \frac{G_t^{1-\zeta}}{1-\zeta} \right) \tag{2.1}$$

subject to the budget constraint

$$P_t C_{pt} + q_t B_{pt} + P_t F_b = P_t w_t L_{pt} + \left(1 + i_{t-1}^B\right) q_{t-1} B_{pt-1} + P_t N_t + P_t \mathcal{D}_t^{IG} + P_t T_{pt}. \tag{2.2}$$

In equation (2.1), $\beta_p \equiv \frac{1}{1+\rho_p} \in (0,1)$ is the discount factor of parents with ρ_p being their rate of time preference, $\sigma > 0$ is the relative risk aversion coefficient, and $\nu > 0$ is the inverse Frisch elasticity of labor supply. Parents spend $P_t C_{pt}$ for consumption goods and $q_t B_{pt}$ for government bonds. The remaining expenditure is a nominal transfer $P_t F_b$ given to the banks in every period as an equity injection.

On the other hand, parents earn nominal income from labor, $P_t w_t L_{pt}$, and receive interest payments on previous savings, $\left(1 + i_{t-1}^B\right) q_{t-1} B_{pt-1}$. The rest of their income consists of dividend income received from intermediate goods firms, $P_t \mathcal{D}_t^{IG}$, the profits of private banks, $P_t N_t$, and a nominal transfer, $P_t T_{pt}$, received from the government. The lump-sum transfer has two parts: a constant part $\mathcal{T}_p$ which is determined at the steady state so that all households consume the same, and a variable part through which the government earns back part of the interest rate payments on its long-term debt.[1] Specifically,

$$P_t T_{pt} = P_t \mathcal{T}_p - \left(1 + i_{t-1}^B\right) q_{t-1} B_{pt-1} - P_t N_t \tag{2.3}$$

As we will see later in the private bank's problem, the last term in the right-hand side of (2.3) is just the interest payment on government debt received by the bank. This payment is given as dividend to the long-term saving household, which is the assumed owner of the bank. The household takes as given the lump-sum transfer T_{pt}. The optimality conditions are:

[1] This assumption is made only for tractability, so as not to carry over the state variable B_{pt-1} in the log-linearization process when deriving the dynamic IS curve, which then allows for analytical solutions.

$$\mu_L L_{pt}^{\nu} = C_{pt}^{-\sigma} w_t \tag{2.4}$$

$$1 = \beta_p \mathbb{E}_t \left[\frac{C_{pt+1}^{-\sigma}}{C_{pt}^{-\sigma}} \frac{P_t}{P_{t+1}} \left(1 + i_t^B\right) \right]. \tag{2.5}$$

Equation (2.4) is the familiar labor supply optimality condition. Equation (2.5) is the familiar Euler equation for long-term government bonds. In equilibrium the consumption function of the long-term savers becomes

$$P_t C_{pt} = P_t w_t L_{pt} + P_t \mathcal{D}_t^{IG} + P_t \mathcal{T}_p - q_t B_{pt} - P_t F_b. \tag{2.6}$$

2.2 Short-term Savers

Short-term savers, or children, constitute the second type of households. The size of the children cohort is $\eta_c \in \left(0, 1 - \eta_p\right)$. Short-term savers discount the future differently, at a rate $\beta_c \in (0,1)$ and derive utility from consumption C_{ct} and disutility from labor L_{ct}. They save by holding only one-period bank deposits D_{ct}, and receive a government transfer T_{ct} which adjusts to keep the amount of real debt constant. Their objective is to pick optimally consumption, labor supply, and bank deposits in each period to maximize lifetime utility

$$\max_{\{C_{ct}, L_{ct}, D_{ct}\}_{t=0}^{\infty}} U_c = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_c^t \left(\frac{C_{ct}^{1-\sigma} - 1}{1-\sigma} - \mu_L \frac{L_{ct}^{1+\nu}}{1+\nu} + \mu_G \frac{G_t^{1-\zeta}}{1-\zeta} \right) \tag{2.7}$$

subject to the budget constraint

$$P_t C_{ct} + D_{ct} = P_t w_t L_{ct} + \left(1 + i_{t-1}^D\right) D_{ct-1} + P_t T_{ct}. \tag{2.8}$$

The optimality conditions for the child are given by

$$\mu_L L_{ct}^{\nu} = C_{ct}^{-\sigma} w_t \tag{2.9}$$

$$1 = \beta_c \left(1 + i_t^D\right) \mathbb{E}_t \left(\frac{C_{ct+1}^{-\sigma}}{C_{ct}^{-\sigma}} \frac{P_t}{P_{t+1}} \right). \tag{2.10}$$

These two previous equations describe the optimal labor supply and the optimal saving behavior of the short-term savers.

2.3 Hand-to-Mouth Households

Hand-to-mouth consumers constitute the last category of households. The size of their cohort is given by η_h, so that $\eta_p + \eta_c + \eta_h = 1$. These households work and consume their income from labor and a constant transfer of T_h from the government. They are excluded from financial markets, so they cannot save by holding government bonds, and they do not enter into any transactions with the banks. They choose consumption C_{ht} and labor L_{ht} to maximize lifetime utility

$$\max_{\{C_{ht}, L_{ht}\}_{t=0}^{\infty}} U_h = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_h^t \left(\frac{C_{ht}^{1-\sigma} - 1}{1-\sigma} - \mu_L \frac{L_{ht}^{1+\nu}}{1+\nu} + \mu_G \frac{G_t^{1-\zeta}}{1-\zeta} \right) \tag{2.11}$$

subject to

$$P_t C_{ht} = P_t w_t L_{ht} + P_t T_h. \tag{2.12}$$

The transfer T_h is such that $C_h = C_p = C_c$ so that all households consume the same amount at the steady state.[2] The optimal labor supply decision for the hand-to-mouth consumers is given by

$$\mu_L L_{ht}^{\nu} = C_{ht}^{-\sigma} w_t. \tag{2.13}$$

2.4 Final Goods Producers

Firms in the final good sector produce the final good Y_t using as inputs a continuum of intermediate goods Y_{jt} with $j \in [0, 1]$. Final good producers operate under perfect competition. They take as given the prices of inputs and choose optimally the quantities of intermediate goods to maximize profits

$$\max_{Y_{jt}} \mathcal{D}_t^F = P_t Y_t - \int_0^1 P_{jt} Y_{jt} dj \tag{2.14}$$

subject to

$$Y_t = \left(\int_0^1 Y_{jt}^{\frac{\varepsilon_p - 1}{\varepsilon_p}} dj \right)^{\frac{\varepsilon_p}{\varepsilon_p - 1}}, \tag{2.15}$$

where P_{jt} is the price of intermediate good j. The solution to the above problem is standard and gives the demand for intermediate good j

[2] The focus of the study is on the differential responses of the households after fiscal shocks and not on steady-state differences.

$$Y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{-\varepsilon_p} Y_t. \tag{2.16}$$

2.5 Intermediate Goods Producers

A continuum of monopolistically competitive firms in the intermediate goods sector is indexed by $j \in [0,1]$. Each firm j produces its own intermediate good Y_{jt} using a linear production technology

$$Y_{jt} = L_{jt}. \tag{2.17}$$

The government subsidizes the firm's total cost at rate τ^L to offset the steady-state distortion associated with monopolistic competition. The total cost of the firm is

$$\mathcal{C}_{jt} = \left(1 - \tau^L\right) w_t L_{jt} = \text{MC}_t Y_{jt}. \tag{2.18}$$

where $\text{MC}_t = \left(1 - \tau^L\right) w_t$ is the real marginal cost of production. It is well-known that the subsidy that eliminates the distortions created by the monopoly power at the steady state satisfies $\tau^L = \frac{1}{\varepsilon_p}$. At this point, the marginal product of labor equals the real wage, so $w = 1$.

Intermediate goods producers optimally pick the price of their products subject to price adjustment costs à la Rotemberg (1982). The price adjustment cost is proportional to the level of final good production. Then they return their profits to the long-term savers who get the profits of these firms. As a result, each intermediate good producer discounts future values and cash flows using the stochastic discount factor of the parents $Q_{t,t+1} \equiv \beta_p \frac{C_{pt+1}^{-\sigma}}{C_{pt}^{-\sigma}}$. The optimal pricing problem of the intermediate good producer j is

$$V_{jt}\left(P_{jt-1}\right) = \max_{P_{jt}} \left\{ \frac{P_{jt}}{P_t} Y_{jt} - \text{MC}_t Y_{jt} - \frac{\xi_p}{2}\left(\frac{P_{jt}}{P_{jt-1}} - 1\right)^2 Y_t + \mathbb{E}_t\left[Q_{t,t+1} V_{jt+1}\left(P_{jt}\right)\right] \right\}. \tag{2.19}$$

The optimality condition for the previous problem, when combined with the equilibrium condition $P_{jt} = P_t$, gives rise to the New Keynesian Phillips Curve

$$\left(1 - \varepsilon_p\right) + \varepsilon_p \text{MC}_t + \mathbb{E}_t\left[Q_{t,t+1} \xi_p \left(\Pi_{t+1} - 1\right) \Pi_{t+1} \frac{Y_{t+1}}{Y_t}\right] = \xi_p \left(\Pi_t - 1\right) \Pi_t, \tag{2.20}$$

where $\Pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate. The profit of each intermediate good producer in equilibrium is

$$\mathcal{D}_t^{IG} = Y_t \left[1 - \text{MC}_t - \frac{\xi_p}{2}\left(\frac{P_t}{P_{t-1}} - 1\right)^2\right]. \tag{2.21}$$

2.6 Private Banks

In the model economy, there is also a banking sector. Banks are essential to unconventional liquidity measures because the central bank's policy partly works through their balance sheets. Specifically, a bank is born in each period and exits with probability one in the next period. The balance sheet of the bank is

$$M_{bt} + q_t B_{bt} = P_t F_b + D_{bt}, \tag{2.22}$$

where M_{bt} denotes the bank's nominal reserve holdings at the central bank, $q_t B_{bt}$ denotes long-term nominal government debt held by the bank, and D_{bt} denotes nominal deposits supplied by the bank. Also, $P_t F_b$ is the bank's equity injection from the long-term savers. Nominal bank profits are given by the difference between interest rate payments received on assets and interest paid on liabilities

$$P_t N_t = \left(1 + i^M_{t-1}\right) M_{bt-1} + \left(1 + i^B_{t-1}\right) q_{t-1} B_{bt-1} - \left(1 + i^D_{t-1}\right) D_{bt-1}. \tag{2.23}$$

The bank's objective is to maximize the accumulated profits over its one period of existence discounted by the stochastic discount factor for nominal payoffs $\Lambda_{t,t+1} \equiv \beta_p \frac{C^{-\sigma}_{pt+1}}{C^{-\sigma}_{pt}} \frac{P_t}{P_{t+1}}$ of the parent household:

$$\max_{M_{bt}, B_{bt}} \mathbb{E}_t \left(\Lambda_{t,t+1} P_{t+1} N_{t+1}\right) = \mathbb{E}_t \left[\Lambda_{t,t+1} \left(i^M_t - i^D_t\right) M_{bt} + \Lambda_{t,t+1} \left(i^B_t - i^D_t\right) q_t B_{bt} + \Lambda_{t,t+1} \left(1 + i^D_t\right) P_t F_b\right] \tag{2.24}$$

subject to a leverage constraint given by

$$q_t B_{bt} \leq \Theta P_t F_b. \tag{2.25}$$

Given this leverage constraint, the bank's holdings of government bonds in each period cannot exceed a multiple of the constant real part of the equity the bank receives from the parent households. I assume that Θ is a constant, so there are no shocks to the leverage constraint. The optimality conditions for the bank's problem are

$$\left(i^M_t - i^D_t\right) \mathbb{E}_t \Lambda_{t,t+1} = 0 \tag{2.26}$$

$$\mathbb{E}_t \Lambda_{t,t+1} \left(i^B_t - i^D_t\right) = \lambda_t. \tag{2.27}$$

Given that $\mathbb{E}_t \Lambda_{t,t+1} > 0$, equation (2.26) implies that $i^M_t = i^D_t$ so that the bank will accumulate reserves up to the point where the interest rate on reserves is equal to the interest rate on deposits. However, the same is not true for government bonds because, on the one hand, the leverage constraint may bind. According to equation (2.27), the average return on government bonds will exceed the deposit rate when the leverage constraint binds. Using the equality $i_t^M = i_t^D$, the bank's transfer to the long-term saving household equals the discounted excess interest payment on government bonds received by the bank plus the discounted interest payment on equity.

2.7 Monetary Authority

The central bank is conducting monetary policy by changing the interest rate on reserves i_t^M and the amount of nominal reserves held by private banks M_t. It also holds a portfolio of long-term government bonds $q_t B_t^{CB}$ as part of its overall monetary policy. The central bank budget constraint is

$$q_t B_t^{CB} + \left(1 + i_{t-1}^M\right) M_{t-1} + P_t T_t^{CB} = M_t + \left(1 + i_{t-1}^B\right) q_{t-1} B_{t-1}^{CB}, \tag{2.28}$$

where $P_t T_t^{CB}$ is a nominal lump-sum transfer the monetary authority makes to the fiscal authority. Policy rules describe the decisions of the central bank. The nominal rate on reserves is set according to a Taylor rule

$$1 + i_t^M = (1 + \rho_c) \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\Pi} \left(\frac{Y_t}{Y}\right)^{\phi_Y}, \tag{2.29}$$

where ρ_c is the rate of time preference of the short-term savers who invest in deposits. The parameters ϕ_Π and ϕ_Y measure how strongly the central bank responds to inflation and output when setting the interest rate on reserves. Also, Π and Y denote steady-state inflation and output, respectively. The amount of real long-term debt bought by the central bank follows a Taylor-type rule as well

$$\frac{q_t B_t^{CB}}{P_t} = \frac{q B^{CB}}{P} \left(\frac{\Pi_t}{\Pi}\right)^{-\psi_\Pi} \left(\frac{Y_t}{Y}\right)^{-\psi_Y}, \tag{2.30}$$

where $\psi_\Pi > 0$ and $\psi_Y > 0$ and $\frac{qB^{CB}}{P}$ is the central bank's long-run target for real asset purchases. The creation of new nominal reserves finances the portfolio of nominal government bonds so that

$$q_t B_t^{CB} = M_t. \tag{2.31}$$

Equations (2.30) and (2.31) imply that the central bank issues more reserves and buys more debt when output falls, or inflation falls. Hence, reserve creation and asset purchase policies are countercyclical.

Finally, given the equality of asset purchases and reserves in equation (2.31), the central bank balance sheet (2.28) simplifies, and the central bank's transfer to the fiscal authority $P_t T_t^{CB}$ can be expressed in each period as a function of the difference between previous payments on assets and liabilities of the central bank

$$P_t T_t^{CB} = \left(1 + i_{t-1}^B\right) q_{t-1} B_{t-1}^{CB} - \left(1 + i_{t-1}^M\right) M_{t-1}. \tag{2.32}$$

The central bank transfers its profit to the government each period. By (2.32), we have that $M_{t-1} = q_{t-1} B_{t-1}^{CB}$, so the transfer will be positive when the leverage constraint faced by the private banks binds since in that case the interest rate on government bonds exceeds the short-term policy rate, and zero if the leverage constraint does not bind.

2.8 Fiscal Authority

The government obtains revenues from issuing new long-term debt, and also from the transfer paid by the monetary authority. Part of the government revenues is used to subsidize employment. Another part is used to finance the lump-sum transfers paid to the different types of households. Government revenues also finance an exogenous path of nominal government expenditures $P_t G_t$ and the interest rate payments on previous debt.

The specification adopted for long-term debt follows Woodford (2001). In any period T, the government issues $\mathcal{I}_T$ nominal perpetual bonds and borrows $q_T \mathcal{I}_T$ in monetary units. Then, it promises to pay $\gamma^s \mathcal{I}_T$ monetary units in every future period $T + s$ for $s > 0$, implying a geometrical decline of coupon payments at rate γ. Thus, we can write the government debt stock in any period t as

$$B_t = \mathcal{I}_t + \gamma \mathcal{I}_{t-1} + \gamma^2 \mathcal{I}_{t-2} + ... + \gamma^{t-1} \mathcal{I}_1 + \gamma^t \mathcal{I}_0 + B_0, \tag{2.33}$$

where B_0 is an exogenous initial condition. According to equation (2.33) in period 0 the government issues $\mathcal{I}_0$ perpetual bonds which pay $\gamma^t \mathcal{I}_0$ in period t. Similarly, the perpetual bonds issued in period 1 pay $\gamma^{t-1} \mathcal{I}_1$ in period t. In the same way, we can sum all the discounted payments implied by issuances up to period t and get the stock of government debt in period t. Equation (2.33) can be written recursively

$$B_t = \mathcal{I}_t + \gamma B_{t-1}. \tag{2.34}$$

The government budget constraint is therefore

$$q_t B_t + P_t T_t^{CB} = P_t \tau^L w_t L_t + \eta_p P_t T_{pt} + \eta_h P_t T_h + \eta_c P_t T_{ct} + (1 + \gamma q_t) B_{t-1} + P_t G_t. \tag{2.35}$$

The government targets a constant amount of real debt $\frac{qB}{P}$ in each period t and adjusts the lump-sum transfer T_{ct} to achieve that. Equation (2.35) implies that the nominal interest rate on government debt is

$$1 + i_{t-1}^{B} = \frac{1 + \gamma q_t}{q_{t-1}}. \tag{2.36}$$

Government expenditures follow an exogenous $AR(1)$ process, subject to an i.i.d. zero-mean shock:

$$G_t = G_{t-1}^{\rho_G} \exp\left(u_t^G\right). \tag{2.37}$$

2.9 Aggregate Equilibrium Conditions

We now turn to the equilibrium of the model. In equilibrium, households, firms, and banks optimize, and the monetary and fiscal authorities follow their rules. The market clearing conditions are

$$D_{bt} = \eta_c D_{ct} \tag{2.38}$$

$$M_t = M_{bt} \tag{2.39}$$

$$B_t = \eta_p B_{pt} + B_{bt} + B_t^{CB} \tag{2.40}$$

$$Y_t = C_t + G_t + \frac{\xi_p}{2}\left(\frac{P_t}{P_{t-1}} - 1\right)^2 Y_t. \tag{2.41}$$

Equation (2.38) is the market clearing condition for deposits. The amount of deposits issued by private banks must equal the amount of deposits held by short-term savers. Equation (2.39) is the reserve market clearing condition and says that the private banks hold all the reserves issued by the central bank. Equation (2.40) is the market clearing condition for government debt and says that the supply of government bonds equals the demand for government debt from the long-term savers, the private banks, and the central bank. Finally, equation (2.41) is the resource constraint of the economy.

The dynamic IS curve of the economy can be obtained by combining the log-linear equilibrium conditions of households, the private banks, and the central bank with the log-linear market clearing conditions.

Proposition 1: *The dynamic IS curve has the following form*

$$\hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \Gamma_g \left(\mathbb{E}_t \hat{g}_{t+1} - \hat{g}_t\right) - \Xi_c \left(i_t^M - \mathbb{E}_t \pi_{t+1} - \rho_c\right) - \Xi_p \left(\mathbb{E}_t \hat{m}_{t+1} - \hat{m}_t\right). \tag{2.42}$$

Proof: See Appendix A.1. ■

In equation (2.42) any variable $\hat{x}_t$ denotes percentage deviations of X_t from its steady state value X. Inflation is also defined as $\pi_t \equiv \ln P_t - \ln P_{t-1}$. The dynamic IS curve summarizes the optimal behavior of all types of households and private banks. It is augmented with fiscal policy and unconventional monetary policy through reserve creation. If there are no hand-to-mouth consumers, so that $\eta_h = 0$, no QE, and no government spending, so that $\hat{m}_t = \hat{g}_t = 0$ for every t, then we get the dynamic IS curve of the textbook three-equation New Keynesian model.

The first three terms on the right-hand side of (2.42) are standard when fiscal policy is present. The new element, though, is the expected difference in the amount of real reserves. Aggregate income increases with the amount of real reserves in period t and decreases with the amount of expected real reserves in period $t+1$. To understand this, we need to focus on the behavior of the long-term savers because the quantitative easing channel works through them. When the central bank purchases more debt in real value at time t, given that the supply of real debt is constant, two things happen: the price of debt increases, and some of the other agents holding debt reduce their bond holdings to restore equilibrium in the bond market. A higher debt price in period t implies a lower expected return on debt, as equation (2.36) suggests. Government bonds become less attractive investments, and long-term savers reduce their debt holdings and consume more at time t, boosting aggregate consumption and income. Moreover, a higher price for bonds relaxes the leverage constraint of the private banks, which allows them to also hold more debt as long-term savers offload their holdings. On the other hand, if the long-term savers expect the central bank to increase its debt purchases in the next period, they expect the price of the debt in the next period to be high, which implies a higher expected return at time t. Government bonds become more attractive, and the long-term savers consume less and buy more bonds[3] at time t, leading to lower aggregate consumption and lower aggregate income.

The production side is not different from the benchmark three-equation New Keynesian model, so the New Keynesian Phillips Curve is derived by log-linearizing equilibrium condition (2.20)

$$\pi_t = \beta_p \mathbb{E}_t \pi_{t+1} + \kappa \widehat{\mathrm{mc}}_t. \tag{2.43}$$

The system is closed with the log-linearized policy rules

$$i_t^M = \rho_c + \phi_\Pi \pi_t + \phi_Y \hat{y}_t \tag{2.44}$$

$$\hat{m}_t = -\psi_\Pi \pi_t - \psi_Y \hat{y}_t \tag{2.45}$$

$$\hat{g}_t = \rho_G \hat{g}_{t-1} + u_t^G. \tag{2.46}$$

[3]The savings decision of the parents takes place before they pay the lump-sum tax and the transfer to the bank. So, when deciding about savings, they consider the expected return of the bonds at time $t+1$.

2.10 Calibration

Households: The value of the parent discount factor that shows up in the NKPC is set equal $\beta_p = 0.997$, following Eggertsson (2011), while the discount factor of the short-term savers is set equal to $\beta_c = 0.99$, which is a standard value. The relative risk aversion coefficient is set equal to $\sigma = 2$, which is also a common value. The inverse of the elasticity of intertemporal substitution in public good consumption is set equal to $\zeta = \sigma = 2$. Following the micro literature, I let $\nu = 2$. The size of the hand-to-mouth consumers' cohort is set equal to $\eta_h = 0.13$, as in McKay et al. (2016), and the size of the short-term savers' cohort is set equal to equal to $2/3$ of the remaining fraction of saving households $1 - \eta_h$ as in Sims et al. (2021). Then, the cohort size of long-term savers is residually determined and equal to $1/3$ of $1 - \eta_h$.

Firms: The value of $\kappa = 0.00859$ is taken from Eggertsson (2011). The value of the price adjustment cost parameter ξ_p is residually determined as follows. The elasticity of substitution between intermediate goods is set equal to $\varepsilon_p = 9$ so that steady state markup equals 12.5%. Then, given the values of κ and ε_p, we can solve for the adjustment cost parameter ξ_p using the definition $\kappa \equiv \frac{\varepsilon_p - 1}{\xi_p}$, which implies $\xi_p \approx 931.32$.

Monetary Authority: The parameters appearing in the monetary policy rule for the nominal interest rate on reserves are set equal to $\phi_\Pi = 1.5$ and $\phi_Y = 0.25$. Those are typical values used in the literature. Next, I follow Sims and Wu (2021) and I assume that the parameters governing the amount of bonds purchased by the central bank are a multiple $7\times$ of the parameters in the Taylor rule for the nominal interest rate on reserves, so $\psi_\Pi = 10.5$ and $\psi_Y = 1.75$. The value of the ratio of steady-state government debt held by the central bank to steady-state total consumption expenditure is set to 10%, the average of the period 1970-2020.

Fiscal Authority: The value of the steady-state government spending-to-output ratio is $g = 23.02\%$, the average between 1970 and 2020. I set $\rho_G = 0.9$, a common value in the literature. Table 1 summarizes the parameter values and the respective targets.

Table 1: Baseline Calibration of Parameter Values

Parameter	Description	Value	Target/Source
Households			
β_p	Parent's Discount Factor	0.997	Standard Value
β_c	Children Discount Factor	0.99	Standard Value
β_c^{ZLB}	Child's Discount Factor at the ZLB	1.005	Real Rate Decline of 2% p.a.
σ	Relative Risk Aversion Coefficient	2	Standard Value
ν	Inverse of Frisch Elasticity	2	Micro Studies
ζ	Inverse of EIS in Public Good Consumption	2	Standard Value
η_h	Hand-to-Mouth Household Size	0.13	McKay et al. (2016)
η_c	Child Household Size	0.58	Sims et al. (2021)
η_p	Parent Household Size	0.29	Residually Determined
Firms			
κ	Degree of Price Stickiness	0.00859	Eggertsson (2011)
ε_p	Elasticity of Substitution Interm. Goods	9	Steady State Markup of 12.5%
ξ_p	Price Adjustment Cost Magnitude	931.32	Residually Determined
Monetary Authority			
ϕ_Π	Inflation Coefficient - Interest Rate Rule	1.5	Standard Value
ϕ_Y	Output Coefficient - Interest Rate Rule	0.25	Standard Value
ψ_Π	Inflation Coefficient - QE Rule	10.5	Sims and Wu (2021)
ψ_Y	Output Coefficient - QE Rule	1.75	Sims and Wu (2021)
ρ_M	Persistence in Asset Purchases	0.9	Baseline Scenario
qB^{CB}/PC	CB-Held Debt to Consumption Expenditure	10%	Average Value 1970-2020
qB^{CB}/PY	CB-Held Debt to GDP	6.56%	Average Value 1970-2020
Fiscal Authority			
ρ_G	Persistence in Government Spending	0.9	Standard Value
g	Government Spending-to-Output Ratio	23.02%	Average Value 1970-2020
Other Parameters			
δ	Probability of Remaining at the ZLB	0.8	Baseline Scenario
wL/C	Labor Compensation to Real Consumption Ratio	0.953	Average Value 1970-2020

2.11 The Fiscal Multiplier and its Determinants

We can derive the fiscal multiplier by exploiting the linear structure of the model and using the method of undetermined coefficients. The following proposition establishes the existence of the fiscal multiplier.

Proposition 2: *If government spending is the only state variable, then the fiscal multiplier on impact is given by the following expression:*

$$\mathcal{M}_0 = \frac{1}{g} \frac{\Gamma_g (1-\rho_G) + \left[\Xi_c (\phi_\Pi - \rho_G) + \Xi_p \psi_\Pi (1-\rho_G)\right] \frac{\kappa}{(1-\beta_p \rho_G)} \frac{\sigma g}{1-g}}{(1-\rho_G)\left(1+\Xi_p \psi_Y\right) + \Xi_c \phi_Y + \left[\Xi_c (\phi_\Pi - \rho_G) + \Xi_p \psi_\Pi (1-\rho_G)\right] \frac{\kappa}{1-\beta_p \rho_G} \left(\frac{\sigma}{1-g} + \nu\right)} \tag{2.47}$$

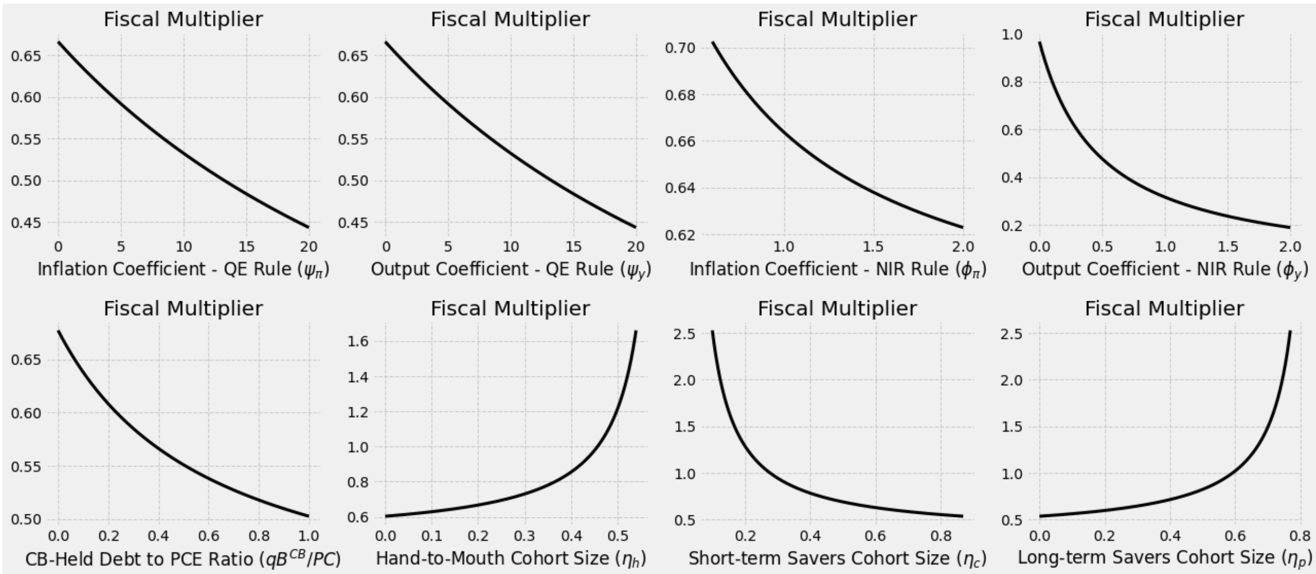
Proof: See Appendix A.2. ■

Using the parameter values in Table 1, $\mathcal{M}_0 = 0.638$, which is a number within the range of values usually reported in the literature. The question at this point is how various parameters of the model affect the size of the fiscal multiplier. The idea is to see how the multiplier behaves when a parameter takes values in a specific range while the other parameters remain constant at their calibrated values. The parameters in the utility function and the NKPC have been discussed extensively in the literature. So, the following paragraphs focus on policy parameters and parameters unique to this model, such as the size of a household type. The corresponding graphs are included in Figure 1.

QE Coefficients: The multiplier is decreasing in the two QE parameters ψ_Π and ψ_Y. The intuition is the following. As aggregate income or inflation increases after a positive fiscal shock, the central bank decreases the amount of reserves and lowers its government bond holdings. The price of the bonds falls, and their expected return increases. The long-term savers find the bonds more attractive, increase their bond holdings and decrease their consumption. As a result, the increase in aggregate consumption is not as high. Higher values for these two parameters imply that the QE response will be stronger; thus, the negative effect on aggregate consumption will be magnified.

Interest on Reserves Coefficients: The multiplier depends negatively on the parameters measuring the central bank's response to output and inflation when deciding the nominal interest rate (NIR) on reserves. This result is well-known and continues to hold in this preferred habitat model. The intuition is that when ϕ_Π and ϕ_Y increase, the central bank responds more strongly to inflation and output, so the increase in the nominal interest rate after a positive fiscal shock will be more significant, and this will lead to a higher real interest rate. This, in turn, discourages consumption today but offers incentives for more short-term savings. As a result, aggregate income today will increase less in response to fiscal action.

Figure 1: Fiscal Multiplier - Comparative Statics



Notes: The various panels depict the fiscal multiplier on impact as a function of different parameters. In all cases the multiplier is computed numerically when one of the parameters takes values in a specific range while the rest remain at their calibrated values.

CB-Held Debt to Consumption Expenditure Ratio: The multiplier depends negatively on the ratio qB^{CB}/PC, so a higher long-term target for real asset purchases or a lower consumption level, in the long run, makes the fiscal policy less effective. This ratio is related to the consumption elasticity with respect to the amount of real asset purchases from the central bank $qB^{CB}/\eta_p PC$. Focusing on the long-term policy ratio qB^{CB}/P, the intuition is that if the central bank has a higher target for its debt holdings, then it will hold a higher amount of government debt on average as equation (2.30) dictates, which implies a lower amount of debt held by long-term savers. Thus, after a positive fiscal shock that leads to a reduction in the amount of reserves, the long-term savers will increase their debt holdings by more since the central bank has room to reduce its debt holdings if needed significantly, and they can increase their debt holdings. So, the consumption of long-term savers will fall by more, and the negative impact on the multiplier will be more significant.

Size of the Hand-to-Mouth Cohort: The values for the size of the hand-to-mouth consumers' cohort considered satisfy $0 \leq \eta_h \leq 0.54$ since for higher values of η_h determinacy is violated, and the model produces negative multipliers. The multiplier is increasing and convex in the size of the hand-to-mouth cohort. Specifically, it starts from 0.623 when $\eta_h = 0$ and increases relatively slow for low values of η_h. Then it becomes larger than 1 for $\eta_h \approx 0.397$.[4] Then, as η_h gets closer to $\eta_h \approx 0.54$, the multiplier becomes large. The significant increase in the value of the multiplier is related to the high *MPCs* of the hand-to-mouth consumers.

[4] This is in contrast with Christiano et al. (2011) who argue that when using a utility function specification as the one I adopt here for all types of households the multiplier is always less than 1. They study a representative agent model.

More constrained households means that the average MPC in the economy increases, so the impact of their increased consumption behavior after a positive demand shock becomes stronger, increasing the multiplier.

Size of the Parents Cohort: The multiplier is increasing in the size of the long-term savers' cohort η_p. The effect mainly driving this result works through the elasticity of the consumption function of the long-term savers with respect to the amount of real reserves $qB^{CB}/\eta_p PC$. As the long-term savers' cohort increases in size, this elasticity falls. Thus, long-term savers will reduce their consumption by less and increase their savings by less after a positive fiscal shock that leads to a reduction in the amount of real reserves and a decrease in the price of bonds. So the overall positive fiscal effect on aggregate consumption will be higher.

Size of the Children Cohort: The fiscal multiplier is decreasing in the size of the short-term savers' cohort η_c. The reason is that having more short-term savers in the economy leads to a more considerable reduction in aggregate consumption after a positive fiscal shock since the expansion of output will lead to an increase in the short-term nominal and real interest rates. As a result, more households will be tempted to reduce their consumption and increase their savings with more short-term savers.

2.12 The Quantitative Easing Multiplier and its Determinants

The model allows the derivation of a quantitative easing multiplier in addition to the fiscal multiplier by allowing for discretionary choices with respect to asset purchases. Let the real amount of reserves follow the following $AR(1)$ process instead of the Taylor rule assumed so far

$$\hat{m}_t = \rho_M \hat{m}_{t-1} + u_t^M. \tag{2.48}$$

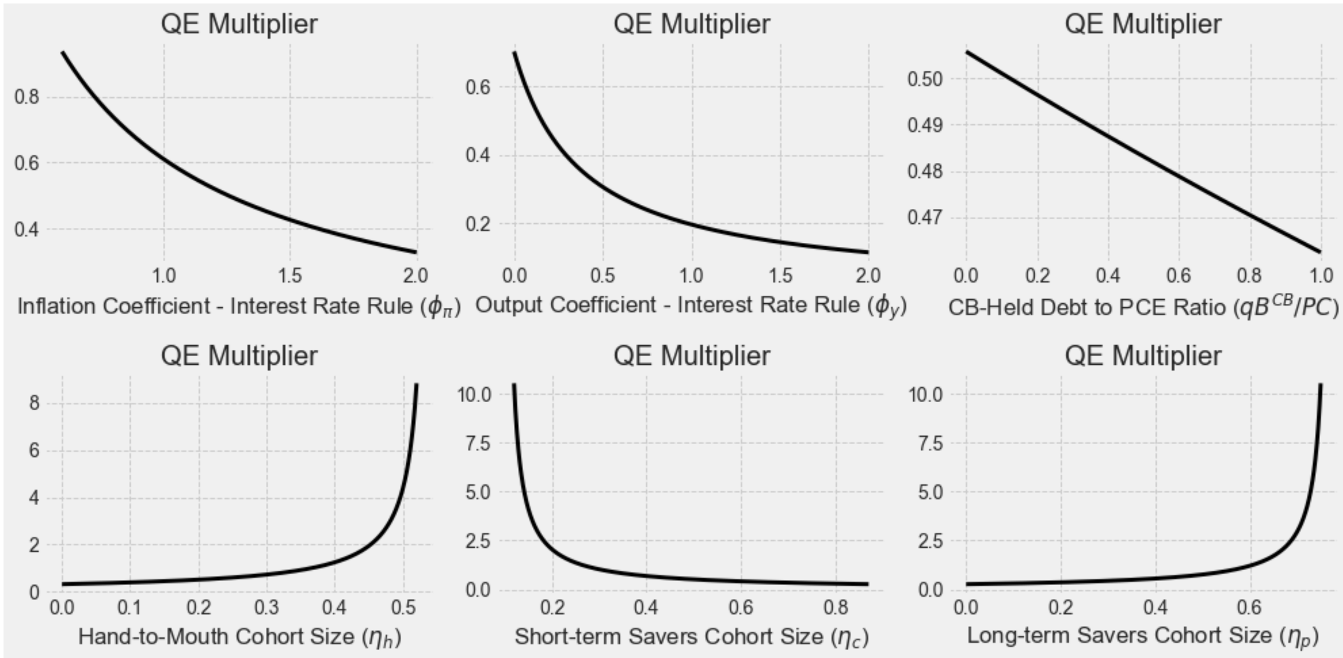
Proposition 3: *If government spending and real reserves are the state variables, then the fiscal and quantitative easing multipliers satisfy*

$$\mathcal{M}_0^G = \frac{1}{g} \frac{\Gamma_g \left(1-\rho_G\right) + \Xi_c \left(\phi_\Pi - \rho_G\right) \frac{\kappa}{\left(1-\beta_p \rho_G\right)} \frac{\sigma g}{1-g}}{1-\rho_G + \Xi_c \phi_Y + \Xi_c \left(\phi_\Pi - \rho_G\right) \frac{\kappa}{1-\beta_p \rho_G} \left(\frac{\sigma}{1-g} + \nu\right)} \tag{2.49}$$

$$\mathcal{M}_0^M = \frac{1}{m} \frac{\Xi_p \left(1-\rho_M\right)}{1-\rho_M + \Xi_c \phi_Y + \Xi_c \left(\phi_\Pi - \rho_M\right) \frac{\kappa}{1-\beta_p \rho_M} \left(\frac{\sigma}{1-g} + \nu\right)}. \tag{2.50}$$

Proof: See Appendix A.3. ■

Figure 2: Quantitative Easing Multiplier - Comparative Statics



Notes: The various panels depict the QE multiplier on impact as a function of different parameters. In all cases the QE multiplier is computed numerically when one of the parameters takes values in a specific range while the other parameters remain constant at their calibrated values.

In equation (2.50) $m \equiv \frac{M}{PY} = \frac{qB^{CB}}{PY}$. The fiscal multiplier on impact is very similar to the one presented in (2.47). By setting $\psi_Y = \psi_\Pi = 0$ in (2.47), we get the fiscal multiplier in (2.49). The behavior of this fiscal multiplier when changing the model's parameter values is similar to the the behavior of the multiplier studied in the previous section.

The quantitative easing multiplier on impact equals $\mathcal{M}_0^M = 0.426$ when using the parameter values in Table 1. Even though the quantitative easing policy does not require higher taxes to be imposed on households after a positive shock, this number is lower than the fiscal multiplier $\mathcal{M}_0^G = 0.607$. Of course, the main reason is that fiscal policy has direct effects in aggregate demand by increasing immediately the output produced, while quantitative easing works indirectly through the effects on asset prices and returns. Thus, in non-crises times fiscal policy can achieve better outcomes in terms of stimulating the economy. Figure 2 suggests that the comparative statics of the quantitative easing multiplier are qualitatively similar to the comparative statics of the fiscal multiplier presented in Figure 1. The two algebraic expressions (2.49) and (2.50) are similar and give the same behavior for the two multipliers when changing the parameters of interest. The intuition in each case is similar too.

2.13 The Aggregate & Distributional Effects of a Fiscal Shock

The model presented exhibits heterogeneity across households, making it reasonable to examine both the aggregate and redistributive effects of a fiscal expansion. To illustrate these effects, an increase in government spending by one percentage point relative to its steady state value is considered.

Figure 3 displays the IRFs of the aggregate variables after a fiscal expansion under two scenarios: in the first case represented by the solid lines, QE follows the countercyclical Taylor rule given by (2.45) with $\psi_\Pi = 10.5$ and $\psi_Y = 1.75$, while in the second scenario, represented by the dashed lines, it is assumed that $\psi_\Pi = \psi_Y = 0$, so asset purchases are kept constant. Under the first scenario, output, inflation, and the real wage rise on impact due to higher aggregate demand. However, profits increase as well because the central bank's countercyclical response leads to a lower increase in the real wage, and the marginal cost increase is not so significant to turn profits negative. This is another way to resolve the countercyclical profits puzzle after fiscal expansions.

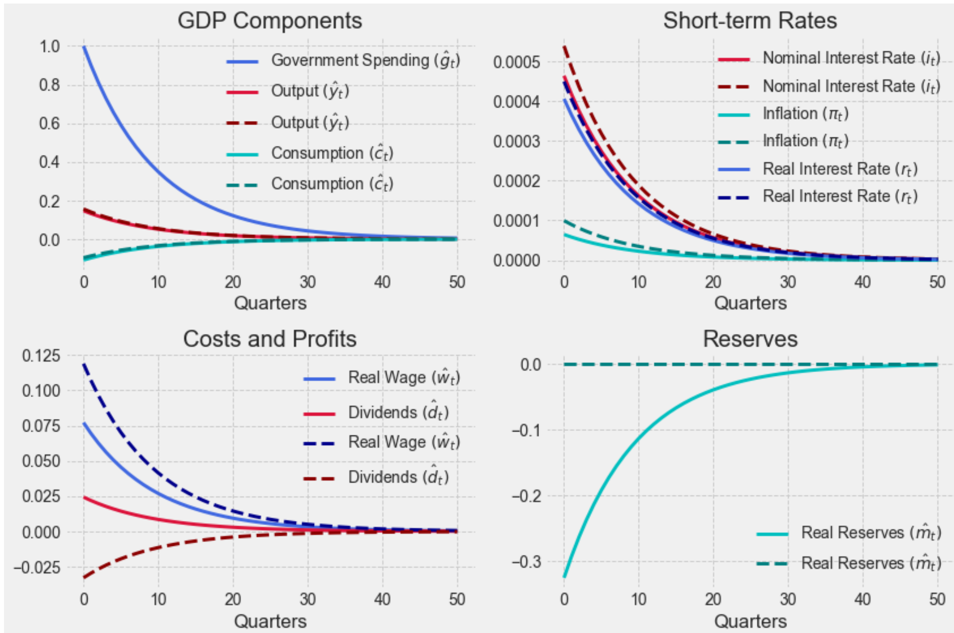
The nominal interest rate follows the movement in output and inflation due to the assumed Taylor rule (2.44). Since the central bank's reaction to inflation is more than one-to-one, the increase in the short-term policy rate on impact is more significant than the increase in inflation, resulting in a higher real interest rate. This offers incentives for higher short-term savings. On the other hand, the amount of reserves decline and follows the opposite path from output and inflation. Aggregate consumption falls on impact and reverts slowly to its steady state value over time.

Under the no QE scenario, we see that all positive effects on output, inflation, the nominal and the real interest rates, and the real wage, are larger. In contrast, the negative effect on aggregate consumption is smaller. Real dividends are lower and negative due to higher wages. However, it is clear from the graph that the differences in the IRFs of output and consumption under the two scenarios are not large. This is because the QE channel in the dynamic IS curve works through the expected difference in the real amount of reserves and not through the actual level of reserves in a specific period, as it happens with the nominal interest rate at time t in the traditional short-term savings channel.

The distributional effects of the fiscal shock are not clear from the behavior of aggregate consumption in Figure 3 since different households decide differently. While aggregate consumption falls with a fiscal expansion, some households can increase their consumption after the shock. The following three equations summarize the consumption functions of the three types of households in equilibrium[5]

[5] These consumption functions are formally derived in Appendix A.1.

Figure 3: Aggregate IRFs to a 1% Shock in Government Spending



Notes: The various panels depict the IRFs of the aggregate variables to a 1% point shock in government spending in the case where QE is countercyclical with $\psi_\Pi = 10.5$ and $\psi_Y = 1.75$ (solid lines), and also when QE is not implemented, when $\psi_\Pi = 0$ and $\psi_Y = 0$ (dashed lines).

$$\hat{c}_{ht} = \frac{(\nu+1)\dfrac{wL}{C}}{\nu + \sigma\dfrac{wL}{C}}\left(\sigma\hat{c}_t + \nu\hat{y}_t\right) \tag{2.51}$$

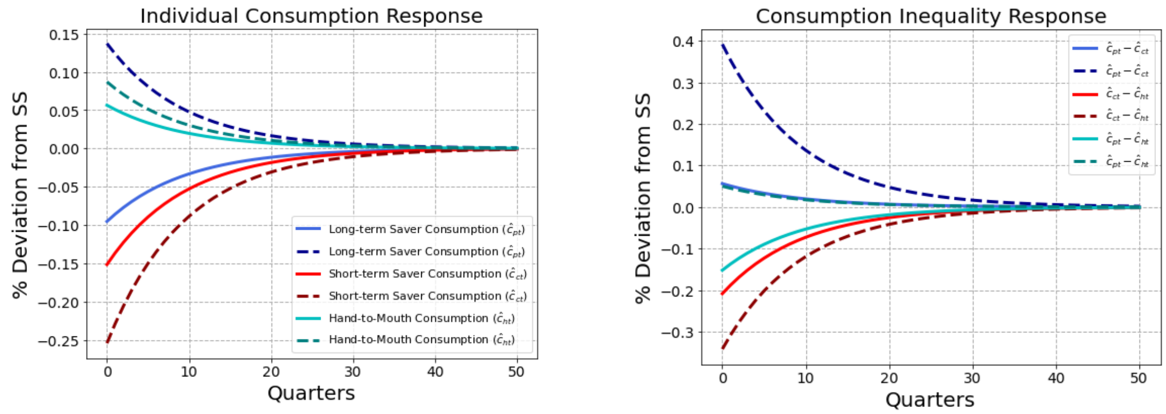
$$\hat{c}_{pt} = \frac{\frac{wL}{C}\frac{\nu+1}{\nu}\left(\frac{\sigma}{1-g}+\nu\right)+\frac{Y}{C}\left[1-w\left(\frac{\sigma}{1-g}+\nu+1\right)\right]}{1+\frac{\sigma}{\nu}\frac{wL}{C}}\hat{y}_t - \frac{\frac{1}{\nu}\frac{wL}{C}\frac{\sigma g}{1-g}}{1+\frac{\sigma}{\nu}\frac{wL}{C}}\hat{g}_t + \frac{qB^{CB}}{\eta_p PC}\frac{1}{1+\frac{\sigma}{\nu}\frac{wL}{C}}\hat{m}_t \tag{2.52}$$

$$\hat{c}_{ct} = \frac{1}{\eta_c}\left(\hat{c}_t - \eta_p\hat{c}_{pt} - \eta_h\hat{c}_{ht}\right). \tag{2.53}$$

Equation (2.51) says that in equilibrium, the consumption of a constrained household depends positively on aggregate consumption and aggregate income. Equation (2.52) gives the consumption function of a long-term saver as a positive function of aggregate income and the amount of real reserves but a negative function of government spending. Finally, equation (2.53) gives the consumption function of a short-term saver. This is residually determined using the expression for aggregate consumption and the individual consumption functions of the other two types of households.

The consumption IRFs for each group of households are plotted in Figure 4. Hand-to-mouth households increase their consumption during the fiscal shock due to higher labor demand, which leads to an increase in

Figure 4: Individual IRFs to a 1% Shock in Government Spending



Notes: The left panel depicts the individual consumption IRFs to a 1% point shock in government spending in the case where QE is countercyclical with $\psi_{\Pi} = 10.5$ and $\psi_Y = 1.75$ (solid lines), and also when QE is not implemented, when $\psi_{\Pi} = 0$ and $\psi_Y = 0$ (dashed lines). The right panel depicts the consumption differences between households under the same two scenaria.

real wages. Using the consumption function (2.51), it is possible to decompose the effects on the consumption of constrained households. When the fiscal shock hits, aggregate consumption falls because short-term savers decrease their consumption due to increased lump-sum taxes needed to finance the fiscal shock. On the other hand, the increase in aggregate income exceeds the reduction in aggregate consumption, and $\hat{c}_{ht}$ rises. Nonetheless, we can deduce that countercyclical QE policies do not help hand-to-mouth consumers since, without QE, the rise in aggregate income and real wages is larger. In that case, the consumption level of the hand-to-mouth increases by more on impact and remains higher during the transition to steady state.

Long-term savers are the first category of households that lower their consumption after the fiscal shock. From equation (2.52), we can see that their consumption is affected positively by aggregate output due to higher labor income and higher sales revenues coming from the firms, negatively by government spending due to higher marginal costs of the firms, and positively by the amount of real asset purchases, because when the central bank increases its asset purchases, bond prices are high, and long-term savers tend to save less and consume more. Under the countercyclical QE scenario, the negative effect of lower real reserves dominates, and the long-term savers lower their consumption. In contrast, under the no QE scenario, the positive effects dominate, and long-term savers' consumption increases.

The last category of households is short-term savers. These households decrease their consumption after the positive fiscal shock, even if their labor income is higher, because the government imposes a higher lump-sum tax on them to keep the government debt constant. In addition, the central bank increases the short-term policy rate after the fiscal expansion, which motivates the short-term savers to hold more deposits, implying higher savings and even lower consumption. During the adjustment periods, the government spending shock gradually fades, and lump-sum taxes decrease over time, so the short-term savers consume more. When the central bank keeps the amount of reserves constant, the reduction in children's consumption after the fiscal shock is higher. This is because the real interest rate is higher without QE, as Figure 3 suggests, which provides extra incentives to short-term savers for higher savings and lower consumption.

The above analysis implies that at the time of the fiscal shock, the households that gain in terms of consumption are the constrained households, while the short-term savers and the long-term savers consume less. The constrained households can increase their consumption relative to the short-term savers, as evident from the red lines in the right panel of Figure 4. Countercyclical quantitative easing weakens this effect since the real interest rate is lower, and short-term savers consume more. In contrast, the real wage is lower, and hand-to-mouth households consume less.

The constrained households can also increase their consumption relative to long-term savers, as the cyan line in the right panel of Figure 4 suggests since long-term savers consume less due to the countercyclical QE response of the central bank. However, this is different under the no QE regime, where the bond holdings of the central bank remain constant and long-term savers increase their consumption by more relative to the constrained households.

Finally, the fiscal shock leads to higher consumption for long-term savers relative to short-term savers since short-term savers reduce their consumption by more than long-term savers. The countercyclical response of the central bank makes this effect weaker due to the lower real interest rate and the reduction in the amount of reserves.

2.14 The Fiscal Multiplier at the Zero Lower Bound

The effects of fiscal policy at the zero lower bound are of particular interest because fiscal policy can be very effective in this case. In particular, if monetary policy pegs the nominal policy rate to zero, then as long as a positive fiscal shock creates expectations for higher inflation in the future, the real interest rate will decline. This, in turn, will create motives for higher consumption and fewer savings at the time of the shock leading to an even higher increase in aggregate output. Of course, the question is whether adopting the countercyclical Taylor rule for QE makes fiscal policy significantly less effective at the ZLB. The ZLB constraint usually binds after strong negative shocks that lead to big recessions and significant cuts in the policy rate. This is precisely what happened during the pandemic and the Great Recession. Such situations create high social costs due to high unemployment, for instance, and call for fiscal actions. Hence, it is essential to know how the effectiveness of fiscal actions is affected by procyclical or countercyclical asset purchasing programs because this will determine whether the social costs of the adverse shocks are alleviated or magnified.

Consider a discount factor shock that leads the economy to the ZLB similar to Christiano et al. (2011). Specifically, let the discount factor of the short-term savers be now stochastic taking two values, the steady state value $\beta_c \in (0,1)$ and $\beta_c^{ZLB} > 1$. This is equivalent to saying that the short-term savers' time preference rate takes the values $\rho_c > 0$ and $\rho_c^{ZLB} < 0$. The following transition probabilities describe the evolution of the stochastic rate of time preference:

$$\begin{aligned} Pr\left(\rho_{ct+1} = \rho_c^{ZLB} \,|\, \rho_{ct} = \rho_c^{ZLB}\right) = \delta \Rightarrow Pr\left(\rho_{ct+1} = \rho_c \,|\, \rho_{ct} = \rho_c^{ZLB}\right) = 1-\delta \\ Pr\left(\rho_{ct+1} = \rho_c^{ZLB} \,|\, \rho_{ct} = \rho_c\right) = 0 \Rightarrow Pr\left(\rho_{ct+1} = \rho_c \,|\, \rho_{ct} = \rho_c\right) = 1 \end{aligned} \tag{2.54}$$

The economy is initially at the steady state, and the discount factor shock hits, making the rate of time preference of the short-term savers equal to ρ_c^{ZLB}. After this period, the evolution of the discount factor follows the process described by (2.54). In each period, the rate of time preference remains negative with probability δ or returns to its steady state value with probability $1-\delta$ and stays at this value permanently.

Monetary policy is now described by a modified for the ZLB Taylor rule, which dictates that the nominal policy rate cannot fall to a level lower than zero:

$$i_t^M = \max\left\{\rho_c + \phi_\Pi \pi_t + \phi_Y \hat{y}_t,\, 0\right\}, \tag{2.55}$$

where in the previous equation $\phi_\Pi > 1$ and $\phi_Y \geq 0$. The shock considered here is supposed to be sufficiently large to make short-term savers save significantly more and consume less, leading to a recession that drives the nominal interest rate to zero. Fiscal policy, on the other hand, is such that the government picks $\hat{g}_t = \hat{g}^{ZLB}$ while the economy is experiencing the discount factor shock, namely for periods $0 \leq t \leq T$, where T is the random final date. For any period $t > T$ the government chooses $\hat{g}_t = 0$.

Given that the discount factor and government spending both return to their steady-state values after period T, then the other variables of the system will also return to their steady-state values, that is $\pi_t = \hat{y}_t = \hat{m}_t = 0$ for $t > T$. The following proposition establishes the existence of the fiscal multiplier at the ZLB.

Proposition 4: *If government spending is the only state variable, the fiscal multiplier at the zero lower bound is given by the following expression*

$$\mathcal{M}_0^{ZLB} = \frac{1}{g} \frac{\Gamma_g (1-\delta) + \left[\Xi_p \psi_\Pi (1-\delta) - \delta \Xi_c\right] \dfrac{\kappa}{(1-\beta_p \delta)} \dfrac{\sigma g}{1-g}}{(1-\delta)\left(1+\Xi_p \psi_Y\right) + \left[\Xi_p \psi_\Pi (1-\delta) - \delta \Xi_c\right] \dfrac{\kappa}{1-\beta_p \delta} \left(\dfrac{\sigma}{1-g} + \nu\right)}. \tag{2.56}$$

Proof: See Appendix A.4. ■

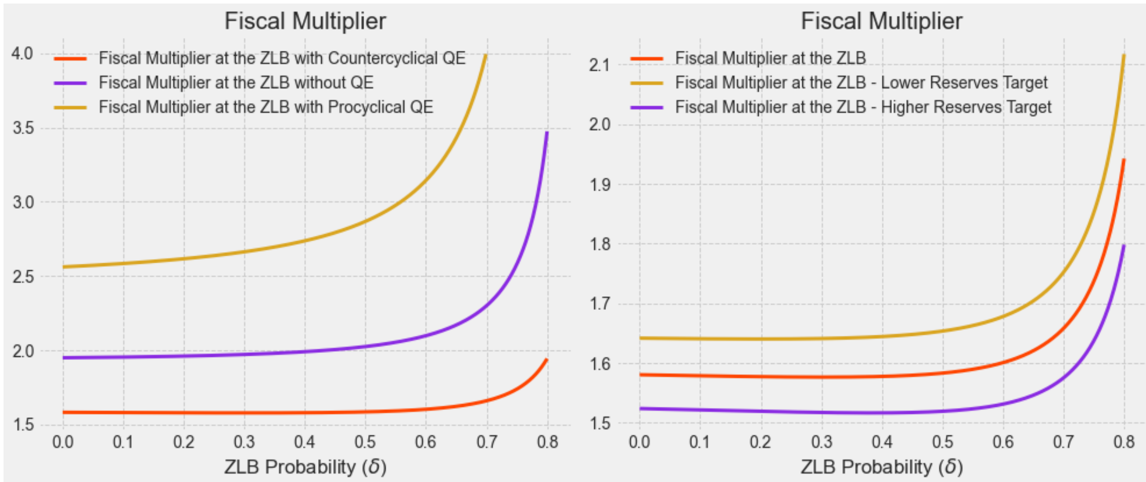
The fiscal multiplier on impact at the ZLB is very similar to the one derived in the general case outside of the ZLB. They differ only with respect to the coefficients ϕ_Π, ϕ_Y, and ρ_G. By setting $\phi_\Pi = \phi_Y = 0$ and replacing ρ_G with δ in (2.47) one gets the expression in (2.56). The problem with setting the coefficients of the monetary policy rule equal to zero is the potential of indeterminacy, except if δ is such that determinacy is ensured. Given the calibrated parameters in Table 1, determinacy is ensured as long as $\delta < 0.82$. Figure 5 plots the fiscal multiplier for values of δ for which determinacy is ensured under different scenarios for the QE parameters. In both panels of Figure 5, the curves corresponding to the baseline calibration are depicted in blue. The other curves correspond to changes in policy parameters. Both panels suggest that a higher probability of remaining at the ZLB in the next period increases the fiscal multiplier implying that fiscal policy is more effective when the ZLB constraint is expected to bind for a longer period. This finding is consistent with what other studies in the past have found. The probability of remaining for one more period at the ZLB significantly affects the value of the multiplier. As it gets closer to 0.82, the multiplier becomes highly sensitive to small changes in δ and takes high values.

Figure 5 shows the relation between the fiscal multiplier and the probability of remaining at the ZLB for one more period. In the left panel, the fiscal multiplier is depicted in three cases: first, when the monetary authority implements QE policies with $\psi_\Pi = 10.5$ and $\psi_Y = 1.75$ reacting countercyclically to a positive fiscal shock; second, when it does not react to the fiscal shock and keeps the amount of reserves constant with $\psi_\Pi = \psi_Y = 0$; third when it reacts procyclically with $\psi_\Pi = -10.5$ and $\psi_Y = -1.75$. It is evident that under a procyclical Taylor rule for reserves, the fiscal multiplier at the ZLB is always higher for any value of δ. This is because monetary policy is now accommodative, and the central bank purchases more bonds after a fiscal expansion. The long-term interest rates fall further, and long-term savers save less and consume more, so the overall effect on aggregate demand is more substantial. Therefore, even though it seems natural to expect that monetary policy reacts countercyclically to various shocks to keep inflation stable and economic activity close to potential, in a liquidity trap where fiscal actions are needed, a countercyclical reaction from the central bank would set hurdles on fiscal policy and would slow down economic recovery.

On the other hand, the right panel depicts the fiscal multiplier when the ratio of long-run assets held by the central bank to consumption expenditure is equal to $qB^{CB}/PC = 10\%$, and also when it is equal to $qB^{CB}/PC = 8\%$ and $qB^{CB}/PC = 12\%$, assuming changes in the long-run target for assets held by the central bank by 20%. The fiscal multiplier is always lower when the central bank holds more assets on average and always higher when it holds fewer assets on average. This is related to long-term savers' consumption function, which depends positively on the amount of real reserves with an elasticity equal to $qB^{CB}/\eta_p PC$ as argued before.

The fiscal multiplier on impact at the ZLB in the three-agent model can be calculated using the expression in (2.56). For the liquidity trap scenario, I consider a 50 basis points increase of the quarterly discount

Figure 5: Fiscal Multiplier and ZLB Probability



Notes: The left panel depicts the fiscal multiplier on impact as a function of the ZLB probability under different scenaria for the QE response of the central bank. The right panel depicts the same relation under various scenaria for the long-term reserves target of the central bank.

factor relative to 1 so that $\beta_c^{ZLB} = 1.005$. This shock is sufficient to lead the economy into a liquidity trap. Also, $\delta = 0.8$, so the probability of remaining at the ZLB is kept as high as possible without exceeding the determinacy limit of 0.82 and simultaneously without getting extremely high fiscal multipliers inconsistent with the empirical studies. After reporting the numbers for the three-agent model, I also compute the multipliers for other models nested in this framework. All the calculated values are reported in Table 2.

PHANK Model: In the three-agent PHANK model, the parameter values of the baseline calibration imply that the fiscal multiplier on impact, as given by (2.47), has a value of 0.638. The multiplier is slightly larger under the no QE scenario, equal to 0.677, increased by 6.11%. The QE channel does not significantly affect the fiscal multiplier in normal times. This is because this channel works through the expected difference in real reserves in the dynamic IS curve and not through the level of real reserves in a specific period. Another reason is that the nominal interest rate can react and partly offset any changes in output and inflation caused by QE policies. On the other hand, at the ZLB, the fiscal multiplier becomes larger and equal to 1.929 if there is countercyclical QE.

Parent/Child Model: The next nested model is a model with parents and children only, without hand-to-mouth consumers similar to Sims et al. (2021). In this case, $\eta_h \to 0$, $\eta_c \to 2/3$ and $\eta_p \to 1/3$. In this case, the value of the multiplier decreases at the limit to 0.603, and the reason is that without hand-to-mouth consumers, the average marginal propensity to consume is lower. The fiscal multiplier again takes higher values when monetary policy is accommodative. It is equal to 0.628 without QE and 1.323 under an

Table 2: The Fiscal Multiplier in Various Nested Models

Parameter Values	**Model**	**Multiplier Size**
Baseline Calibration	PHANK	0.638
$\psi_\Pi = \psi_Y = 0$	PHANK, No QE	0.677
$\phi_\Pi = \phi_Y = 0$	PHANK, ZLB	1.929
$\eta_h \to 0$	Parent/Child	0.603
$\eta_h \to 0$ & $\psi_\Pi = \psi_Y = 0$	Parent/Child, No QE	0.628
$\eta_h \to 0$ & $\phi_\Pi = \phi_Y = 0$	Parent/Child, ZLB	1.323
$\eta_p \to 0$	TANK	0.548
$\eta_p \to 0$ & $\psi_\Pi = \psi_Y = 0$	TANK, No QE	0.548
$\eta_h \to 0$ & $\phi_\Pi = \phi_Y = 0$	TANK, ZLB	1.648
$\eta_h \to 0$ & $\eta_p \to 0$	RANK	0.527
$\eta_h \to 0$ & $\eta_p \to 0$ & $\psi_\Pi = \psi_Y = 0$	RANK, No QE	0.527
$\eta_h \to 0$ & $\eta_p \to 0$ & $\phi_\Pi = \phi_Y = 0$	RANK, ZLB	1.187

interest rate peg to zero. At the ZLB, the fiscal multiplier is lower than the PHANK model, which shows the importance of agents with high *MPCs* in a situation like a liquidity trap. Given the accommodative monetary policy, the increase in aggregate demand and real wages is higher, so the hand-to-mouth can increase their consumption even more. This effect is lost in the parent/child model.

TANK Model: The third nested model is a TANK model. Here there are no long-term savers, that is $\eta_p \to 0$, $\eta_h = 0.13$ as before, and $\eta_c \to 0.87$. The impact multiplier equals 0.548, which is lower than the parent/child model. The multiplier falls more in this case because now the short-term savers' size has increased, which lowers the aggregate consumption increase after a fiscal shock since these households have the incentive to increase their deposits when interest rates increase in response to the shock. In addition, we see that the multiplier remains the same if there is no QE, and this is because the QE channel works only through the long-term savers that do not exist in the TANK model. The multiplier is again higher under an interest rate peg, equaling 1.648.

RANK Model: Finally, we have the representative agent model in which $\eta_p \to 0$ and $\eta_h \to 0$ and $\eta_c \to 1$. This is the model with the lowest multiplier on impact, equal to 0.527 because there are only short-term savers. Again, when the economy hits the zero lower bound, the multiplier is higher and equal to 1.187.

2.15 The Quantitative Easing Multiplier at the Zero Lower Bound

In recent years, the monetary authorities in the US and Europe have used extensively quantitative easing policies while stuck at the effective lower bound for nominal interest rates. A natural question arising is how effective these policies are. The model introduced so far can answer this question by deriving the quantitative easing multiplier at the zero lower bound. The scenario for the liquidity trap is the same as in subsection 2.14. However, quantitative easing is now decided by the government directly and not through a Taylor rule, so while the economy remains at the ZLB for periods $0 \leq t \leq T$, the policy sector picks $\hat{m}^{ZLB}$. When the economy is out of the liquidity trap for $t > T$, the amount of real reserves returns to its steady state level, implying that $\hat{m}_t = 0$.

Proposition 5: *If government spending and real reserves are the state variables, then the fiscal and quantitative easing multipliers at the zero lower bound satisfy*

$$\mathcal{M}_0^{G,ZLB} = \frac{1}{g} \frac{\Gamma_g (1-\delta) - \delta \Xi_c \dfrac{\kappa}{(1-\beta_p \delta)} \dfrac{\sigma g}{1-g}}{1 - \delta - \delta \Xi_c \dfrac{\kappa}{1-\beta_p \delta} \left(\dfrac{\sigma}{1-g} + \nu \right)} \tag{2.57}$$

$$\mathcal{M}_0^{M,ZLB} = \frac{1}{m} \frac{\Xi_p (1-\delta)}{1 - \delta - \delta \Xi_c \dfrac{\kappa}{1-\beta_p \delta} \left(\dfrac{\sigma}{1-g} + \nu \right)}. \tag{2.58}$$

Proof: See Appendix A.5. ■

The two previous multipliers are equal to $\mathcal{M}_0^{G,ZLB} = 2.788$ and $\mathcal{M}_0^{M,ZLB} = 3.573$ when using the parameter values from Table 1. The quantitative easing multiplier is slightly larger than the fiscal multiplier and significantly greater than one. However, this multiplier corresponds to a policy that does not require the increase in lump-sum taxes paid by short-term savers to keep the real debt constant or, in a more general setting, to keep the lifetime budget constraint of the government balanced. This could provide incentives first to use unconventional monetary policies at the ZLB. Then if these policies fail to bring the economy out of the liquidity trap, fiscal policy can be used to stimulate aggregate demand. In addition, quantitative easing has the advantage that it can be initiated immediately after a negative shock hits the economy, while fiscal policy is subject to legislation lags. As Christiano et al. (2011) have shown, the timing of policy implementation plays a crucial role in fiscal policy effectiveness.

2.16 Optimal Fiscal Policy at the Zero Lower Bound

The fiscal multiplier, at the zero lower bound, can be affected by the reserve policy of the monetary authority. This raises the question about the optimal level of government spending that would maximize social welfare in at the ZLB, given the behavior of the private sector and the behavior of the central bank. In a setup with different agents, such as the one presented here, the government considers each agent's consumption and labor supply levels when deciding about optimal government spending. The government is assumed to be utilitarian, and the social welfare function is the weighted sum of individual utilities, where the corresponding weight for each group equals the group's population size. The following proposition provides an expression for the social welfare function.

Proposition 6: *Suppose that the government is utilitarian so that social welfare is given by $\mathcal{W} = \eta_p U_p + \eta_c U_c + \eta_h U_h$. Let also β_G be the discount factor of the government. A quadratic approximation of the previous social welfare function around the steady state gives rise to the social welfare function*

$$\mathcal{L} = -\frac{U_C Y}{2}\mathbb{E}_0 \sum_{t=0}^{T} \beta_G^t \left[\sigma\left(1-g\right)\left(\eta_p \hat{c}_{pt}^2 + \eta_c \hat{c}_{ct}^2 + \eta_h \hat{c}_{ht}^2\right) + \nu\left(\eta_p \hat{l}_{pt}^2 + \eta_c \hat{l}_{ct}^2 + \eta_h \hat{l}_{ht}^2\right) + g\zeta \hat{g}_t^2 + \xi_p \pi_t^2\right]. \quad (2.59)$$

Proof: See Appendix A.6. ■

The first three terms inside the brackets represent the quadratic approximation to the utility contribution for all agents if prices were the same. The last inflation term represents the cost of price dispersion. The government maximizes the social welfare function subject to the behavior of the private sector and the central bank. At this point it is still assumed that fiscal policy picks $\hat{g}_t = \hat{g}^{*ZLB}$ while the economy is trapped at the zero lower bound for periods $0 \leq t \leq T$, and $\hat{g}_t = 0$ for $t > T$. The economy returns to the steady state equilibrium when the zero lower bound period ends.

Proposition 7: *At the zero lower bound, the social welfare function takes the following form*

$$\mathcal{L}^{ZLB} = -\frac{U_C Y}{2}\frac{1-\left(\beta_G \delta\right)^{T+1}}{1-\beta_G \delta}\left[\sigma\left(1-g\right)\left(\eta_p \hat{c}_p^2 + \eta_c \hat{c}_c^2 + \eta_h \hat{c}_h^2\right) + \nu\left(\eta_p \hat{l}_p^2 + \eta_c \hat{l}_c^2 + \eta_h \hat{l}_h^2\right) + g\zeta \hat{g}^2 + \xi_p \pi^2\right] \quad (2.60)$$

In addition, there exists a unique solution to the government's problem of choosing $\hat{g}^{*ZLB}$ optimally to maximize the above social welfare function subject to the constraints imposed by the behavior of the private sector and the central bank.

Proof: See Appendix A.7. ■

In the above expression, the ZLB notation from the terms on the right-hand side is suppressed to facilitate exposition. The maximization of the previous social welfare function is subject to the households' consumption and labor supply responses, and the dynamic IS curve, the New Keynesian Phillips Curve, the Taylor-type rule for real reserves, and the resource constraint.

Using the parameter values in Table 1, the optimal deviation of government spending from the steady state is $\hat{g}^{*ZLB} = 5.94\%$. This number is significantly lower than 30% found by Christiano et al. (2011). One reason for this significant difference is the existence of hand-to-mouth households and long-term savers whose behavior increases the fiscal multiplier, so the government can choose a smaller increase in government spending to achieve a specific increase in aggregate output. A second reason is the already high calibrated value $g = 23.02\%$, which implies that there is not so much fiscal space that would allow for a higher increase in government purchases without creating further significant taxation burdens to households.

An important question is how the optimal choice $\hat{g}^{*ZLB}$ is affected by the model's parameters. This is critical since it allows the design of better fiscal packages according to the specific characteristics of an economy. The following paragraphs provide the comparative statics analysis that answers this question. The corresponding graphs are summarized in Figure 6.

ZLB Probability: Optimal government spending at the ZLB is an increasing function of the ZLB probability for one more period δ. This is not surprising since if the households believe that the liquidity trap will persist for many periods, they will keep choosing lower consumption levels and higher savings, resulting in a significant recession. To avoid this situation, the government optimally chooses to increase $\hat{g}^{*ZLB}$ when the probability of remaining at the ZLB is higher to drive the economy out of the liquidity trap.

Inflation Coefficient - QE Rule: Optimal government spending at the ZLB depends negatively on the inflation coefficient ψ_{Π}. This is related to the response of the short-term savers to the government spending shock. A higher value of ψ_{Π} implies that the inflation response of the central bank will be stronger after the government spending shock, and expected inflation will rise by less. The real interest rate will be higher and short-term savers will save more and consume less. In addition, their consumption will be lower because the lump-sum transfer they receive will be lower due to the fiscal expansion. Therefore, a stronger negative response of the central bank to inflation leads to higher losses for the short-term savers for a given amount of government spending, and the government chooses to lower $\hat{g}^{*ZLB}$ as ψ_{Π} increases.

Output Coefficient - QE Rule: The optimal government spending amount at the ZLB is first a positive, and then it becomes a negative function of the output coefficient ψ_Y. The logic for the negatively sloped part of the curve is similar to the case of ψ_{Π}. For the positively sloped part, after a positive fiscal shock, a higher value of ψ_Y implies that the countercyclical reaction of the central bank to the increase in output will be stronger, so output and inflation will increase by less. The real interest rate will be higher, and the fiscal multiplier will be lower, so the fiscal expansion will be less stimulative. For that reason, the fiscal authority chooses to increase government purchases by more to provide extra stimulation in economic activity.

Size of the Hand-to-Mouth Cohort: The optimal government spending at the ZLB is first a positive function of the size of hand-to-mouth households, then it reaches a maximum and then becomes a negative function of η_h. Initially, when η_h is small, the government has an incentive to increase $\hat{g}^{*ZLB}$ together with the number of constrained households since these households gain from such a policy. However, as the number of constrained households in the economy grows, a higher amount of government spending leads to higher output and higher inflation. The short-term savers experience more significant losses due to higher taxation, and the long-term savers reduce their consumption by more due to the more considerable decrease in the amount of real reserves implied by the QE rule. Moreover, a higher value of η_h increases the fiscal multiplier, so the government can spend less and substantially increase economic activity. Hence, the government chooses optimally to lower $\hat{g}^{*ZLB}$ if η_h increases.

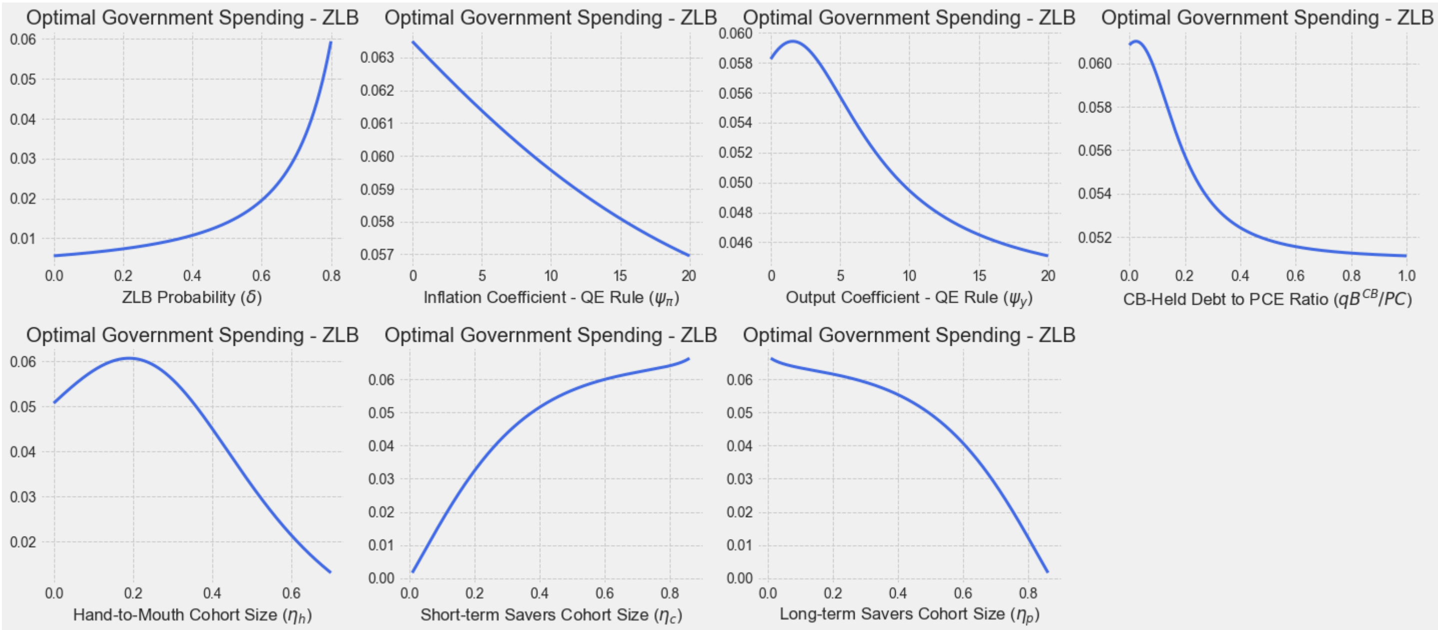
Size of the Parents Cohort: The optimal government spending amount at the ZLB is a negative function of the long-term savers' cohort size η_p. The intuition is the following. As the size of the parent cohort increases, the fiscal multiplier increases, so a lower $\hat{g}^{*ZLB}$ is needed to stimulate the economy and push it out of the liquidity trap.

Size of the Children Cohort: The optimal choice $\hat{g}^{*ZLB}$ decreases when the size of the short-term savers η_c increases because, in that case, the fiscal multiplier is lower, so a higher $\hat{g}^{*ZLB}$ is needed to stimulate the economy and push it out of the liquidity trap.

CB-Held Debt to Consumption Expenditure Ratio: The optimal government spending choice at the ZLB is first a positive function of qB^{CB}/PC, then it reaches a maximum and then becomes a negative function of that ratio. Specifically, for small values of qB^{CB}/PC, given that the fiscal multiplier is a negative function of this ratio, the government needs to spend a higher amount to stimulate aggregate demand and achieve higher economic activity. However, as this ratio becomes larger, the negative effect of quantitative easing policies on the consumption of long-term savers becomes too large, and the government decides optimally to lower $\hat{g}^{*ZLB}$ when qB^{CB}/PC is higher.

Figure 7 depicts $\hat{y}^{ZLB}$, π^{ZLB}, i^M and social welfare $\mathcal{L}^{ZLB}$ as functions of $\hat{g}$. The red vertical line in the graphs corresponds to the optimal choice $\hat{g}^{*ZLB}$ that arises in a situation where the discount factor shock hits the economy and the resulting recession is such that the zero lower bound binds. The orange line corresponds to the choice $\hat{g}^{ZLB}$ that makes the nominal interest rate positive. The government steps in and chooses

Figure 6: Optimal Fiscal Policy at the ZLB - Comparative Statics



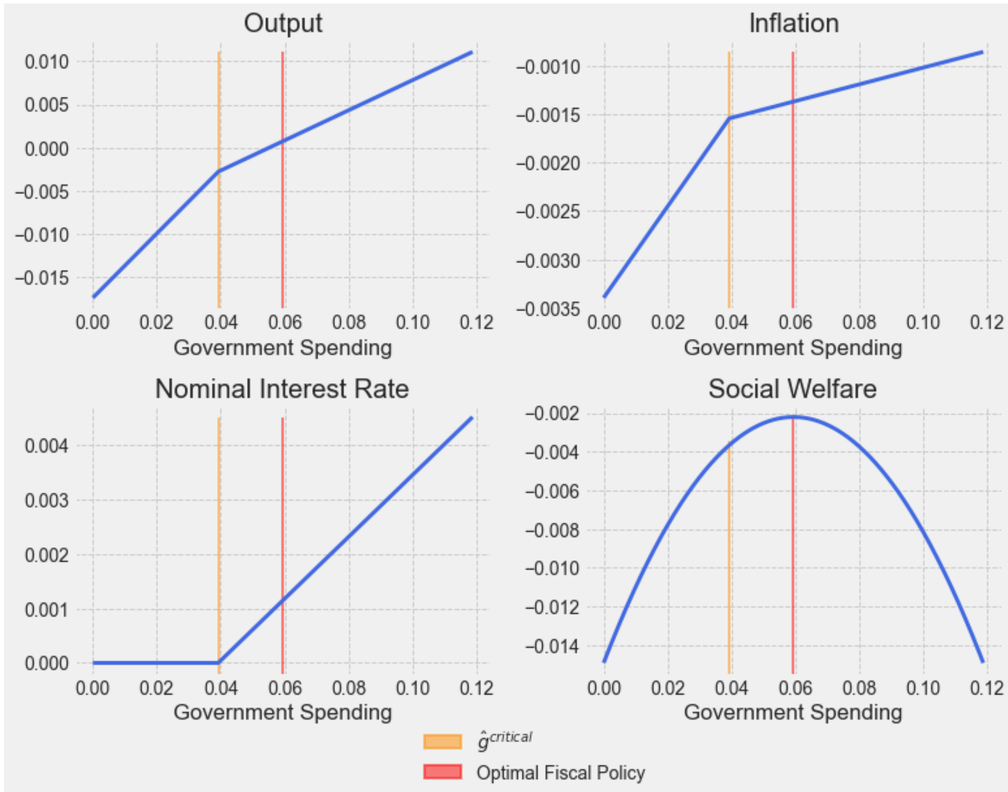
Notes: The various panels depict the optimal level of government spending at the ZLB as a function of different parameters.

optimally to increase the amount of government spending to drive the economy out of the liquidity trap. It is evident from the lower left panel of Figure 7 that the optimal choice $\hat{g}^{*ZLB}$ is sufficient to drive the economy out of the zero lower bound since the nominal interest rate becomes positive again. The reason is that the percentage deviation of output from the steady state becomes positive and is enough to offset the small deflation implied by the upper right panel. At this optimal level of government spending deviation, the fiscal multiplier is actually given by equation (2.47), where ρ_G is replaced by δ, because the economy is out of the ZLB.

2.17 Optimal Fiscal and Quantitative Easing Policies at the Zero Lower Bound

So far, the assumption about quantitative easing policies was that real reserves are determined in each period by a countercyclical Taylor-type rule. This assumption is now dropped, and the governmental sector can simultaneously choose optimally the amount of reserves and the amount of government spending at the zero lower bound. The problem of the government is to choose optimally $\hat{g}^{*ZLB}$ and $\hat{m}^{*ZLB}$ in order to maximize the social welfare function given in (2.60) subject to the constraints imposed by the behavior of the other agents of the economy. However, the Taylor-type rule for reserves is not part of the set of constraints since the amount of reserves is optimally chosen.

Figure 7: Optimal Fiscal Policy at the ZLB



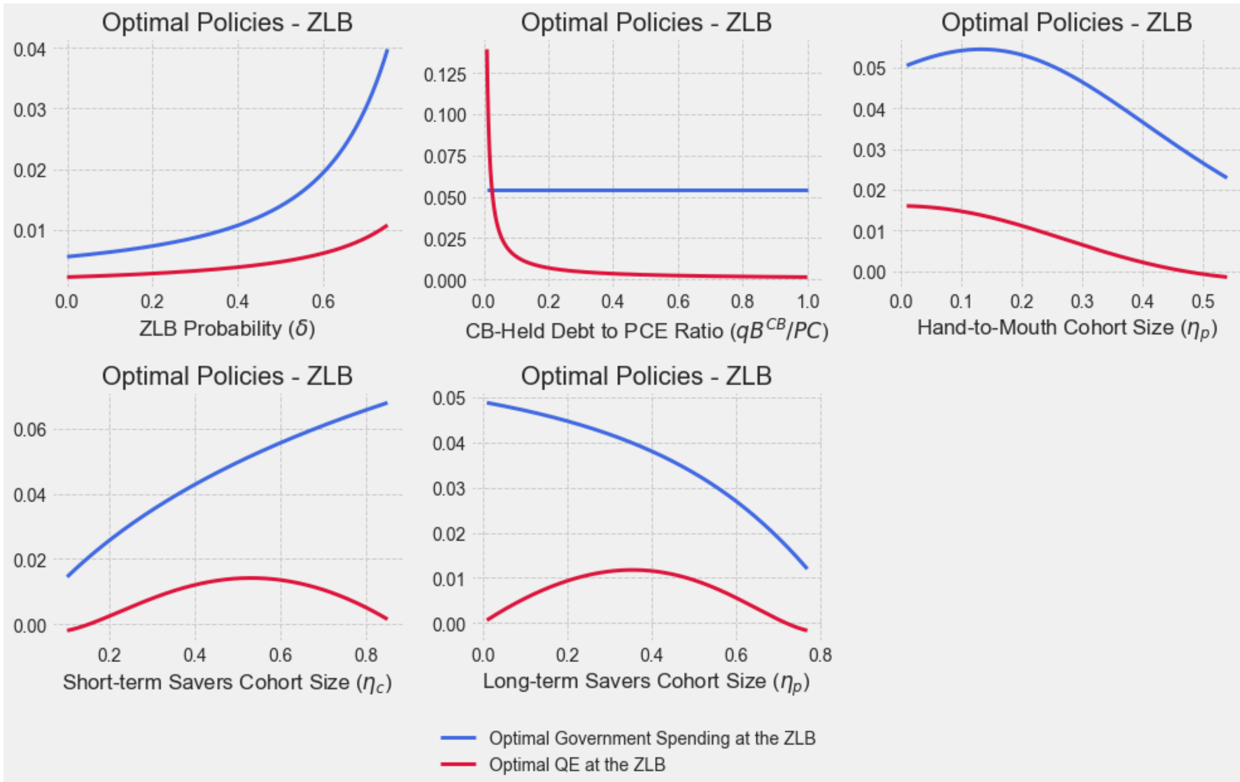
Notes: The various panels depict aggregate variables of interest (blue lines) as functions of government spending. Optimal government spending is depicted in red. The orange vertical line depicts the lowest choice of government spending deviation that drives the economy outside of the ZLB.

Proposition 8: *There exists a unique solution to the government's problem of choosing optimally $\hat{g}^{*ZLB}$ and $\hat{m}^{*ZLB}$ to maximize the social welfare function in (2.60) subject to the constraints imposed by the behavior of the private sector.*

Proof: See Appendix A.8. ∎

Given the parameter values from baseline calibration, the optimal policies are such that $\hat{g}^{*ZLB} = 5.44\%$ and $\hat{m}^{*ZLB} = 1.39\%$. The government chooses optimally to increase government spending and the amount of reserves in a liquidity trap, implying a procyclical behavior for quantitative easing. The increase in government spending is higher than in asset purchases, probably because government spending has a direct effect on output through higher aggregate demand, whereas asset purchases affect output indirectly through the consumption/savings decisions of the long-term savers. The comparative statics analysis for the optimal choice $\hat{g}^{*ZLB}$ is qualitatively similar to the analysis in the previous section, so the focus now will be on the behavior of $\hat{m}^{*ZLB}$ when the parameters of the change. Figure 8 contains the corresponding graphs.

Figure 8: Optimal Fiscal and QE Policies - Comparative Statics

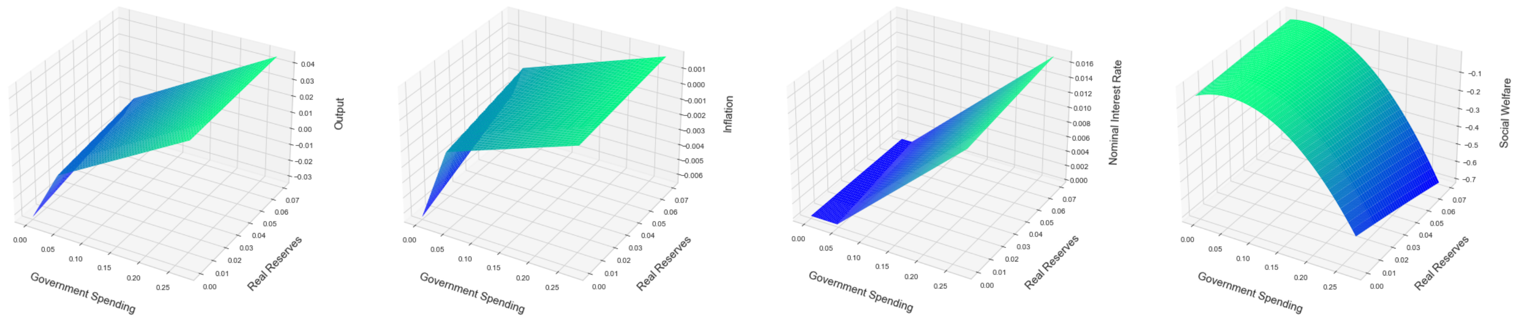


Notes: The various panels depict the optimal level of government spending (black lines) and quantitative easing (maroon lines) at the ZLB as functions of different parameters of interest.

ZLB Probability: The optimal choice of QE at the ZLB is an increasing function of the probability of staying at the ZLB for one more period δ. Again, if the liquidity trap is expected to persist for many periods, then agents will keep choosing low consumption levels and high savings, resulting in a severe recession. The government chooses optimally to increase $\hat{m}^{*ZLB}$ and $\hat{g}^{*ZLB}$ relative to their steady-state values when δ is higher. Interestingly, $\hat{m}^{*ZLB}$ remains always lower than $\hat{g}^{*ZLB}$ since this allows to minimize taxation distortions, and fiscal policy is more effective.

CB-Held Debt to Consumption Expenditure Ratio: The optimal QE choice at the ZLB is a negative function of that ratio. Specifically, as qB^{CB}/PC increases, the optimal choice $\hat{m}^{*ZLB}$ falls fast while $\hat{g}^{*ZLB}$ increases very slowly. A higher ratio qB^{CB}/PC implies that the elasticity of the consumption of long-term savers with respect to real asset purchases will be high, so $\hat{c}_p$ will increase by more after a positive choice $\hat{m}^{*ZLB}$. Therefore, the government needs to increase by less the optimal choice $\hat{m}^{*ZLB}$ to stimulate the economy through the long-term savings channel. Low values of $\hat{m}^{*ZLB}$ lead to higher values of $\hat{g}^{*ZLB}$ to sufficiently stimulate the economy.

Figure 9: Optimal Fiscal and QE Policies at the ZLB



Notes: The various panels depict aggregate variables of interest as functions of optimal government spending and optimal QE. In the lower left panel, the ZLB constraint binds for low levels of government consumption and central bank asset purchases.

Size of the Hand-to-Mouth Cohort: The optimal government spending at the ZLB is first a positive function of the size of hand-to-mouth households, then it reaches a maximum and then becomes a negative function of η_h as before, and the intuition is similar. On the other hand, optimal QE is a negative function of η_h. If the size of the hand-to-mouth households increases, fiscal and QE policies are more effective, so less QE is needed to sufficiently stimulate the economy. Also, with more hand-to-mouth households, less parents and less children exist in the economy given that $\eta_c = \frac{2}{3}(1-\eta_h)$ and $\eta_p = \frac{1}{3}(1-\eta_h)$, which implies that reserves policy is not increasing social welfare significantly, leading to lower $\hat{m}^{*ZLB}$.

Size of the Parents Cohort: The optimal government spending amount at the ZLB is a negative function of the long-term savers' cohort size η_p. The intuition is similar to before. The fiscal multiplier is higher, so lower stimulation is needed. The optimal QE choice is first a positive function of η_p, then it reaches a maximum and becomes a negative function of η_p. Given that $\hat{g}^{*ZLB}$ falls with η_p, and since higher η_p implies a lower elasticity of children consumption with respect to real reserves, a higher amount of QE is needed to stimulate the economy. Nevertheless, if η_p increases too much, fiscal and QE policies become more effective so the government can choose lower $\hat{m}^{*ZLB}$ and $\hat{g}^{*ZLB}$ and still sufficiently stimulate the economy.

Size of the Children Cohort: The optimal choices $\hat{g}^{*ZLB}$ and $\hat{m}^{*ZLB}$ have the opposite behavior with respect to the size of short-term savers η_c since more short-term savers make both policies less effective in terms of stimulating economic activity, which in general incentivizes the government to increase both government spending and asset purchases.

Figure 9 depicts $\hat{y}^{ZLB}$, π^{ZLB}, i^M and $\mathcal{L}^{ZLB}$ as functions of $\hat{g}$ and $\hat{m}$. This time the government deploys both unconventional monetary policy and fiscal policy to stimulate the economy. The optimal choices $\hat{g}^{*ZLB}$ and $\hat{m}^{*ZLB}$ are sufficient to drive the economy out of the ZLB.

3 A Medium-Scale HANK Model

I now present a two-asset HANK model. In the economy, there are heterogeneous households subject to individual productivity shocks and save by holding one liquid and one illiquid asset to insure against these shocks. The model contains typical New-Keynesian frictions, such as nominal rigidities in prices and wages and investment adjustment costs, but is also enhanced with financial frictions to generate differences in the returns of various assets and make QE effective.

3.1 Households

The demand side of the economy consists of infinitely-lived households indexed by $i \in [0,1]$. Households are assumed to be ex-ante identical but ex-post heterogeneous due to the evolution of their idiosyncratic productivity shock e_i, and their different holdings of illiquid and liquid assets, $\tilde{a}_i$ and $\tilde{d}_i$.[6] Each individual i derives utility from consumption c_i and disutility from labor supply l_i. In each period, individuals choose consumption, labor supply, and savings to maximize lifetime utility

$$\max_{\{c_{it}, l_{it}, a_{it}, d_{it}\}_{t=0}^{\infty}} U_i = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_{it}^{1-\sigma} - 1}{1-\sigma} - \mu_L \frac{l_{it}^{1+\nu}}{1+\nu} \right) \tag{3.1}$$

subject to

$$c_{it} + d_{it} + a_{it} + \Xi\left(a_{it}, a_{it-1}\right) + \mathcal{T}_t = \left(1 - \tau^L\right) w_t l_{it} e_{it} + \left(1 + r_t^D\right) d_{it-1} + \left(1 + r_t^A\right) a_{it-1} \tag{3.2}$$

$$\Xi\left(a_{it}, a_{it-1}\right) = \frac{\chi_1}{\chi_2} \left| \frac{a_{it} - \left(1 + r_t^A\right) a_{it-1}}{\left(1 + r_t^A\right) a_{it-1} + \chi_0} \right|^{\chi_2} \left[\left(1 + r_t^A\right) a_{it-1} + \chi_0 \right] \tag{3.3}$$

$$a_{it} \geq 0, \;\; d_{it} \geq \underline{d}. \tag{3.4}$$

Equation (3.1) is the period budget constraint. Household i spends c_{it} for consumption goods, d_{it} for liquid assets, a_{it} for illiquid assets and $\mathcal{T}_t$ for lump-sum taxes. The saving decisions are constrained by the liquidity constraints given in (3.4). The decision for the illiquid asset is subject to a cost of adjustment which takes the form given in equation (3.3) with χ_0, $\chi_1 > 0$ and $\chi_2 > 1$. Note that the adjustment cost function $\Xi\left(a_{it}, a_{it-1}\right)$ is bounded, differentiable, and convex with respect to a_{it}. Assets of type a are considered illiquid since individuals incur a cost to adjust this part of their portfolio.

Individuals finance expenditures for consumption and savings with income earned from various sources. The first source is labor income $\left(1 - \tau^L\right) w_t l_{it} e_{it}$, where w_t denotes the real wage, l_{it} denotes the amount

[6] For any nominal variable $\tilde{X}_t$, the real counterpart is given by $X_t = \frac{\tilde{X}_t}{P_t}$.

of labor supplied which is decided by a labor union for each individual, and e_{it} denotes the productivity shock received by household i. The latter is modeled as a Markov process with transition probability matrix $\Omega\left(e_{t+1}|e_t\right)$. Labor income is subject to a constant linear labor income tax τ^L. Other sources of income are interest payments on previous savings $\left(1+r_t^D\right)d_{it-1}$ and $\left(1+r_t^A\right)a_{it-1}$. The interest rates in the previous terms are determined in equilibrium by the behavior of financial intermediaries. The households take the tax rate τ^L and the lump-sum tax $\mathcal{T}_t$ as given. The fiscal rule behind the lump-sum tax is discussed later when the government is introduced.

3.2 Labor Unions

Labor unions are introduced to allow for sticky wages in the model economy. Sticky wages solve the trilemma problem described in Auclert et al. (2021a) of matching the data on the size of *MPCs*, *MPEs*, and fiscal multipliers. Following Hagedorn et al. (2019b), I assume that each household i provides differentiated labor services $l_{it}e_{it}$ to a labor union, and then the union sells these labor services to a representative and competitive labor recruiting firm. The labor recruiter demands the same differentiated labor as the intermediate goods producers. The problem of the labor recruiter is to minimize the cost of producing a given amount of aggregate labor:

$$\min_{l_{it}} \mathcal{C}_t^{LR} = \int_0^1 W_{it}l_{it}e_{it}di \tag{3.5}$$

subject to the technology constraint

$$L_t = \left[\int_0^1 \left(l_{it}\right)^{\frac{\varepsilon_w-1}{\varepsilon_w}} e_{it}\right]^{\frac{\varepsilon_w}{\varepsilon_w-1}}, \tag{3.6}$$

where ε_w is the elasticity of substitution between differentiated labor services. The demand of the labor recruiter for each differentiated labor service is

$$l_{it} = \left(\frac{W_{it}}{W_t}\right)^{-\varepsilon_w} L_t, \tag{3.7}$$

where in the last equation $W_t = \left[\int_0^1 e_{it}W_{it}^{1-\varepsilon_w}\right]^{\frac{1}{1-\varepsilon_w}}$ is the equilibrium nominal wage. The labor union picks the same nominal wage $W_{it} = \hat{W}_t$ for all households to maximize profits, subject to a wage adjustment cost à la Rotemberg (1982). The cost is proportional to the amount of total effective labor L_t and is denoted as $\Xi^w\left(W_t, W_{t-1}, L_t\right)$. The problem of the union can be expressed as

$$V_t^w\left(\hat{W}_{t-1}\right) \equiv \max_{\tilde{W}_t}\left\{\int \frac{e_{it}\left(1-\tau^L\right)\hat{W}_t}{P_t} l\left(\hat{W}_t; W_t, L_t\right) di - \frac{v\left(l\left(\hat{W}_t; W_t, L_t\right)\right)}{u'\left(C_t\right)}\right.$$
$$\left. - \int \frac{\xi_w}{2}\left(\frac{\hat{W}_t}{\hat{W}_{t-1}}-1\right)^2 L_t e_{it} di + \frac{V_{t+1}^w\left(\hat{W}_t\right)}{1+r_{t+1}}\right\}. \tag{3.8}$$

In the previous equation $V_t^w(.)$ is the value function of the union, r_{t+1} is the real interest rate[7] in period $t+1$ and ξ_w is a parameter that determines the magnitude of the cost of adjusting wages. In the special case that $\xi_w = 0$, wages are fully flexible. In equilibrium $\hat{W}_t = W_t$ and $l_{it} = L_t$. The optimality condition for the union's problem is a Phillips curve relation for wages:

$$\left(1-\tau^L\right)\left(1-\varepsilon_w\right) w_t + \varepsilon_w \frac{v'\left(l\left(W_t, L_t\right)\right)}{u'\left(C_t\right)} + \frac{1}{1+r_{t+1}} \xi_w \left(\Pi_{t+1}^w - 1\right) \Pi_{t+1}^w \frac{L_{t+1}}{L_t} = \xi_w \left(\Pi_t^w - 1\right) \Pi_t^w. \tag{3.9}$$

where in equation (3.9) nominal wage inflation can be written in terms of price inflation and real wage inflation, so that

$$\Pi_t^w = \Pi_t \frac{w_t}{w_{t-1}}. \tag{3.10}$$

3.3 Commercial Banks

The first type of financial intermediaries in the economy is commercial banks. A continuum of commercial banks is indexed by $b \in [0,1]$. Each bank collects part of the households' liquid savings $\tilde{D}_{bt}$ and issues deposits to them. These deposits pay a nominal return $\left(1+i_t^D\right)$ in the next period subject to an exogenous cost of financial intermediation per unit ξ_D that lowers in equilibrium the return on the liquid asset. These deposits are invested in nominal reserves $\tilde{M}_{bt}$ issued by the central bank. The balance sheet constraint of a commercial bank in real terms is

$$M_{bt} = D_{bt} + N_{bt}, \tag{3.11}$$

where N_{bt} is the real net worth of commercial bank b. Net worth evolution is described by the difference between interest rate payments received on previous central bank reserves and the interest paid on households' previous deposits

[7]Since the experiments studied here are perfect foresight experiments after one-time MIT-type aggregate shocks in period $t = 0$, the real interest rate can be used as the global discount factor for every agent different than households.

$$N_{bt} = (1 + r_t) M_{bt-1} - \left(1 + r_t^D + \xi_D\right) D_{bt-1}. \tag{3.12}$$

In the previous equation the real interest rate is given by the Fisher equation

$$1 + r_t = \frac{1 + i_{t-1}^M}{\Pi_t}, \tag{3.13}$$

where i_t^M is the nominal interest rate paid on reserves, and Π_t is the gross inflation rate in period t. A commercial bank's problem is to optimally pick the amount of real reserves M_{bt} to invest in and the amount of real deposits D_{bt} to issue to maximize its value. This is equal to the present discounted value of the weighted average of net worth in the next period and the continuation value in the next period, where the weighting scheme consists of the corresponding probabilities of exiting the market and continuing operating in the market for one more period:

$$V_{bt}(N_{bt}) = \max_{M_{bt}, D_{bt}} \left\{ \frac{1}{1 + r_{t+1}} \left[(1 - \theta_b) N_{bt+1} + \theta_b V_{bt+1}(N_{bt+1}) \right] \right\} \tag{3.14}$$

subject to equations (3.11), (3.12).

Proposition 9: *The commercial bank's value function is linear in net worth and satisfies*

$$V_{bt}(N_{bt}) = \Sigma_t N_{bt} \text{ with } \Sigma_t = 1. \tag{3.15}$$

Proof: See appendix A.9. ■

Using the two constraints (3.11) and (3.12) in equation (3.14), the optimality condition for the private bank is

$$r_{t+1} = r_{t+1}^D + \xi_D. \tag{3.16}$$

Equation (3.16) says that the financial intermediaries will accumulate reserves until the real return on reserves equals the real cost of deposits. Due to the financial intermediation cost, the real return on deposits is lower in equilibrium than the real interest rate. Finally, we can aggregate the individual net worth evolution equation (3.12) over b by taking into account the survival rate θ_b and the fact that new banks, when entering the market, get a transfer ωM_{t-1}. In this way, we get the equation for total net worth evolution written at the end of period t as:

$$N_t = \theta_b \left[(1 + r_t) M_{t-1} - \left(1 + r_t^D + \xi_D\right) D_{t-1} \right] + \omega M_{t-1}. \tag{3.17}$$

The profits of the commercial banking sector are given by

$$\Pi_{bt} = (1-\theta_b)\left[(1+r_t)\,M_{t-1} - \left(1+r_t^D+\xi_D\right)D_{t-1}\right] - \omega M_{t-1}. \tag{3.18}$$

The above expression is just the total net worth of exiting banks minus the resources given to new banks. The government taxes part of the profits of the commercial banks at a rate τ^D. The remaining part is given to the investment fund, which owns the commercial banks.

3.4 Investment Fund

In the economy, a hypothetical investment fund collects the illiquid savings of households A_t and invests these resources in deposits issued by investment banks. The investment fund also owns all firms in the economy and collects all the profits. In equilibrium, it pays a real return to households r_t^A subject to a cost of financial intermediation ξ_A. The balance sheet of the investment fund in real terms is

$$A_t = F_t, \tag{3.19}$$

where F_t is the depoosits of the investment fund in the investment banks. The budget constraint is

$$F_t + \left(1+r_t^A+\xi_A\right)A_{t-1} = \left(1-\tau^D\right)\mathcal{D}_t + A_t + \left(1+r_t^F\right)F_{t-1}, \tag{3.20}$$

where $\mathcal{D}_t$ is the aggregate profits of all firms in the economy taxed at the rate τ^D. The mutual fund is assumed to operate under a no-retained earnings rule. This, together with the balance sheet constraint, imply that in equilibrium, the real return paid to households is determined by

$$r_t^A = \frac{\left(1-\tau^D\right)\mathcal{D}_t}{A_{t-1}} + r_t^F - \xi_A. \tag{3.21}$$

3.5 Investment Banks

The second type of financial intermediary in the home country is investment banks. This is modeled in a way similar to Gertler and Karadi (2011) and Gertler and Karadi (2013). A continuum of financial intermediaries is indexed by $n \in [0,1]$. The banks collect the investment fund's transfer $\mathcal{F}_t$. With these funds available, the investment banks invest in government bonds $\mathcal{B}_{nt}^{MF}$ with price q_t that pay a nominal return $\left(1+i_t^B\right)$, capital claims issued by the intermediate goods firms $P_tK_{nt+1}^{MF}$ with price Q_t.

The balance sheet constraint and the budget constraint of each investment bank is

$$Q_t K_{nt}^{MF} + q_t B_{nt}^{MF} = F_{nt}^{MF} + N_{nt}^{MF} \tag{3.22}$$

$$Q_t K_{nt}^{MF} + q_t B_{nt}^{MF} + \left(1 + r_t^F\right) F_{nt-1}^{MF} + \xi_N N_{t-1} = \left(1 + r_t^K\right) Q_{t-1} K_{nt-1}^{MF} + \left(1 + r_t^B\right) q_{t-1} B_{nt-1}^{MF} + F_{nt}^{MF}. \tag{3.23}$$

where ξ_N is a cost per unit of net worth, which can be thought of as a managerial cost. The combination of the previous two equations gives the evolution of the investment bank's net worth

$$N_{nt}^{MF} = \left[\left(1 + r_t^K\right) - \left(1 + r_t^F\right)\right] Q_{t-1} K_{nt-1}^{MF} + \left[\left(1 + r_t^B\right) - \left(1 + r_t^F\right)\right] q_{t-1} B_{nt-1}^{MF} + \left(1 + r_t^F - \xi_N\right) N_{nt-1}^{MF}. \tag{3.24}$$

In each period, a fraction of banks θ^{MF} continues to operate, while the remaining fraction $1 - \theta^{MF}$ exits the market. The banking sector is characterized by a moral hazard problem. In each period a bank can divert a fraction λ_K of the equity holdings, $Q_t K_{nt}^{MF}$, and a fraction λ_B of the government bonds, $q_t B_{nt}^{MF}$. All the agents involved can force the banker into bankruptcy and recover the rest of the assets $(1 - \lambda_K) Q_t K_{nt}^{MF}$, and $(1 - \lambda_B) q_t^B B_{nt}^{MF}$ respectively, but it is too costly for them to recover the diverted assets. A banker will avoid diverting assets when the value of financial intermediation V_{nt}^{MF} is greater or equal to the value obtained from diverting assets. Putting all the previous together, we can express the bank's problem as follows:

$$V_{nt}^{MF}\left(N_{nt}^{MF}\right) = \max_{K_{nt}^{MF}, B_{nt}^{MF}, F_{nt}^{MF}} \left\{ \frac{1}{1 + r_{t+1}} \left[\left(1 - \theta^{MF}\right) N_{nt+1}^{MF} + \theta^{MF} V_{nt+1}^{MF}\left(N_{nt+1}^{MF}\right)\right] \right\} \tag{3.25}$$

subject to equations (3.22) and (3.23), and the incentive constraint

$$V_{nt}^{MF}\left(N_{nt}^{MF}\right) \geq \lambda_K Q_t K_{nt}^{MF} + \lambda_B q_t B_{nt}^{MF}. \tag{3.26}$$

Proposition 10: *The private bank's value function is linear in net worth and satisfies*

$$V_{nt}^{MF}\left(N_{nt}^{MF}\right) = \Sigma_t^{MF} N_{nt}^{MF} \tag{3.27}$$

$$\Sigma_t^{MF} = \frac{\lambda_K}{\lambda_K - \frac{1}{1+r_{t+1}}\left(1 - \theta^{MF} + \theta^{MF}\Sigma_{t+1}^{MF}\right)\left(r_{t+1}^K - r_{t+1}^F\right)} \frac{1 + r_{t+1}^F - \xi_N}{1 + r_{t+1}} \left(1 - \theta^{MF} + \theta^{MF}\Sigma_{t+1}^{MF}\right). \tag{3.28}$$

Proof: See Appendix A.10. ■

The optimality conditions corresponding to the bank's problem can be combined into a single equation

$$1+r_{t+1}^{B}=\frac{\lambda_B}{\lambda_K}\left(1+r_{t+1}^{K}\right)+\left(1-\frac{\lambda_B}{\lambda_K}\right)\left(1+r_{t+1}^{F}\right). \tag{3.29}$$

Equation (3.30) says that the real return on government bonds in equilibrium will be a weighted average of the real return paid on equity and the real return paid on savings. The weighting scheme is determined by the ratio of the divertible fraction of government debt held by banks to the divertible fraction of equity held by banks $\frac{\lambda_B}{\lambda_K}<1$. Thus, the bank will accumulate government debt until the marginal benefit from the real return on this debt equals the marginal cost of investing in public debt. This cost is partly given by the financing cost from attracting savings and partly by the opportunity cost from not investing these savings in capital. The aggregate incentive constraint determines the amount of capital claims held by the banks

$$K_t^{MF}=\frac{1}{\lambda_K Q_t}\left(\Sigma_t^{MF}N_t^{MF}-\lambda_B q_t B_t^{MF}\right) \tag{3.30}$$

Finally, we can aggregate the individual net worth over n and take into account the survival rate θ^{MF} and the fact that new banks, when entering the market, get a transfer $\omega^{MF}\left(Q_{t-1}K_{t-1}^{MF}+q_{t-1}B_{t-1}^{MF}\right)$ which is a fraction of the total assets held by banks in the previous period. Then, the total net worth is

$$\begin{aligned} N_t^{MF} = {} & \theta^{MF}\left\{\left[\left(1+r_t^{K}\right)-\left(1+r_t^{F}\right)\right]Q_{t-1}K_{t-1}^{MF}+\left[\left(1+r_t^{B}\right)-\left(1+r_t^{F}\right)\right]q_{t-1}B_{t-1}^{MF}+\left(1+r_t^{F}-\xi_N\right)N_{t-1}^{MF}\right\} \\ & +\omega^{MF}\left(Q_{t-1}K_{t-1}^{MF}+q_{t-1}B_{t-1}^{MF}\right) \end{aligned} \tag{3.31}$$

The profits of the investment banks are given by

$$\mathcal{D}_t^{MF}=\frac{1-\theta^{MF}}{\theta^{MF}}N_t^{MF}-\frac{\omega^{MF}}{\theta^{MF}}\left(Q_{t-1}K_{t-1}^{MF}+q_{t-1}B_{t-1}^{MF}\right). \tag{3.32}$$

The above expression is just the total net worth of existing banks minus the funds given to new banks. Part of the profits of the financial sector is taxed at a rate τ^D. The remaining part is given to the investment fund, which owns the banks. The interest rate paid on deposits by the banks to the investment fund is determined by imposing a zero profit condition[8] on the banking sector as a whole.

[8] This condition was chosen since under this assumption the fiscal multipliers outside the ZLB under a Taylor rule take reasonable values. Otherwise the multipliers outside the ZLB can take substantial values.

The zero-profit condition leads to

$$1+r_t^F = \frac{1}{F_{t-1}}\left[\left(1+r_t^K\right)Q_{t-1}K_{t-1}^{MF} + \left(1+r_t^B\right)q_{t-1}B_{t-1}^{MF} - \xi_N N_{t-1} - \frac{\omega^{MF}}{1-\theta^{MF}}\left(Q_{t-1}K_{t-1}^{MF} + q_{t-1}B_{t-1}^{MF}\right)\right]. \tag{3.33}$$

3.6 Final Goods Producers

Firms in the final good sector produce the final good Y_t using as inputs a continuum of intermediate goods Y_t^j with $j \in [0,1]$. Final good producers operate under perfect competition. They take as given the prices of inputs and choose optimally the quantities of intermediate goods to maximize profits

$$\max_{Y_t^j} \mathcal{D}_t^F = P_t Y_t - \int_0^1 P_t^j Y_t^j dj \tag{3.34}$$

subject to

$$Y_t = \left[\int_0^1 \left(Y_t^j\right)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} dj\right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}}, \tag{3.35}$$

where P_t^j is the price of intermediate good j. The solution to the above problem is standard and gives the demand for intermediate good j

$$Y_t^j = \left(\frac{P_t^j}{P_t}\right)^{-\varepsilon_p} Y_t. \tag{3.36}$$

3.7 Intermediate Goods Producers

A continuum of firms in the intermediate goods sector is indexed by $j \in [0,1]$. Each firm j has access to the same CRS production technology, which combines labor L_t^j, capital K_{t-1}^j, and total factor productivity Z_t to produce output Y_t^j:

$$Y_t^j = Z_t \left(K_{t-1}^j\right)^{\alpha} \left(L_t^j\right)^{1-\alpha} \tag{3.37}$$

Intermediate goods producers hire capital and labor in perfectly competitive markets in a way that minimizes the cost of production

$$\min_{K_{t-1}^j, L_t^j} \mathcal{C}_t^j = w_t L_t^j + \left(1+r_t^K\right) Q_{t-1} K_{t-1}^j - \left(Q_t - \delta\right) K_{t-1}^j \tag{3.38}$$

The cost minimization is subject to the production function (3.37). The optimality conditions are

$$r_t^K = \frac{1}{Q_{t-1}}\left(\alpha \frac{Y_t^j}{K_{t-1}^j}\mathrm{MC}_t + Q_t - \delta\right) - 1 \tag{3.39}$$

$$w_t = (1-\alpha)\frac{Y_t^j}{L_t^j}\mathrm{MC}_t \tag{3.40}$$

$$\mathrm{MC}_t = \left(\frac{r_t^K}{\alpha}\right)^{\alpha}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}. \tag{3.41}$$

In the previous equations MC_t is the real marginal cost of production. Total cost is then

$$\mathcal{C}_t^j = \left(1 + r_t^K\right) Q_{t-1}K_{t-1}^j - (Q_t - \delta)\, K_{t-1}^j + w_t L_t^j = \mathrm{MC}_t Y_t^j. \tag{3.42}$$

Intermediate goods producers also pick the price of their products subject to the demand for intermediate inputs from the side of the final good producers as given in (3.36). However, every time they adjust their prices they have to undertake a price adjustment cost à la Rotemberg (1982), which is denoted here as $\Xi_t^P\left(P_t^j, P_{t-1}^j, \Pi_{t-1}, Y_t\right)$. This cost is proportional to the final good. The optimal pricing problem is

$$V_{jt}\left(P_{jt-1}\right) = \max_{P_{jt}}\left\{\left(\frac{P_{jt}}{P_t} - \mathrm{MC}_t\right)\left(\frac{P_{jt}}{P_t}\right)^{-\varepsilon_p} Y_t - \frac{\xi_p}{2}\left(\frac{P_{jt}}{P_{jt-1}} - \Pi_{t-1}^{\zeta}\Pi^{1-\zeta}\right)^2 Y_t + \frac{V_{jt+1}\left(P_{jt}\right)}{1+r_{t+1}}\right\} \tag{3.43}$$

In the previous equation, $\Pi = 1$ is the steady state gross inflation rate. Also, ζ measures how strong the backward-looking behavior of the firms is when setting prices in an equivalent Calvo price-setting setup. In equilibrium, $P_t^j = P_t$, so the optimality condition for the optimal pricing problem gives rise to a New Keynesian Phillips Curve augmented with a backward-looking inflation term

$$(1-\varepsilon_p) + \varepsilon_p \mathrm{MC}_t + \frac{1}{1+r_{t+1}}\xi_p\left(\Pi_{t+1} - \Pi_t^{\zeta}\Pi^{1-\zeta}\right)\Pi_{t+1}\frac{Y_{t+1}}{Y_t} = \xi_p\left(\Pi_t - \Pi_{t-1}^{\zeta}\Pi^{1-\zeta}\right)\Pi_t. \tag{3.44}$$

In the special case $\xi_p = 0$ prices are flexible and real marginal cost is $\mathrm{MC}_t = \frac{\varepsilon_p - 1}{\varepsilon_p}$. The dividend paid by each intermediate good producer is

$$\mathcal{D}_t^{IG} = Y_t\left[1 - \mathrm{MC}_t - \frac{\xi_p}{2}\left(\frac{P_t}{P_{t-1}} - \Pi_{t-1}^{\zeta}\Pi^{1-\zeta}\right)^2\right] \tag{3.45}$$

Part of the dividends is taxed by the government at rate τ^D. The remaining part is given to the investment fund which owns the firms.

3.8 Capital Producers

A representative firm buys the capital stock at the end of any period t, builds new capital to replenish the depreciated capital, and then sells the total amount of capital to the intermediate goods producers for a price Q_t. New capital is built in each period t by undertaking investment I_{nt} subject to increasing and convex investment adjustment costs denoted as $\Xi_t^I (I_{nt}, I_{nt-1})$. The objective of the capital-producing firm is

$$\max_{I_{nt}} \sum_{s=t}^{\infty} \left(\prod_{k=1}^{j|_{j\geq 1}} \frac{1}{1+r_{t+k}} \right) \left[(Q_s - 1) I_{ns} - \frac{\xi_I}{2} \left(\frac{I_{ns}+I}{I_{ns-1}+I} - 1 \right)^2 (I_{ns}+I) \right] \tag{3.46}$$

where new and total investment satisfy the law of motions

$$I_{nt} = I_t - \delta K_{t-1} \tag{3.47}$$

$$I_t = K_t - (1-\delta) K_{t-1}. \tag{3.48}$$

The optimality condition is

$$Q_t + \frac{1}{1+r_{t+1}} \xi_I \left(\frac{I_{nt+1}+I}{I_{nt}+I} - 1 \right) \left(\frac{I_{nt+1}+I}{I_{nt}+I} \right)^2 = 1 + \left[\xi_I \left(\frac{I_{nt}+I}{I_{nt-1}+I} - 1 \right) \frac{I_{nt}+I}{I_{nt-1}+I} + \frac{\xi_I}{2} \left(\frac{I_{nt}+I}{I_{nt-1}+I} - 1 \right)^2 \right]. \tag{3.49}$$

Part of the dividends of the capital producing-firm is taxed by the government at rate τ^D. The remaining part is given to the investment fund which owns the firms.

3.9 Monetary Authority

The central bank is conducting monetary policy by changing the nominal interest rate on reserves i_t^M and purchasing government debt $q_t \tilde{B}_t^{CB}$. The central bank budget constraint in real terms is

$$q_t B_t^{CB} + (1+r_t) M_{t-1} + T_t^{CB} = \left(1 + r_t^B\right) q_{t-1} B_{t-1}^{CB} + M_t, \tag{3.50}$$

where T_t^{CB} is the real lump-sum transfer between the monetary and fiscal authorities. Taylor-type rules describe the behavior of the central bank. The nominal rate on reserves is given by

$$1 + i_t^M = \max \left\{ (1+r)^{1-\rho} \left(1 + i_{t-1}^M\right)^{\rho} \left(\frac{\Pi_t}{\Pi} \right)^{(1-\rho)\phi_\Pi} \left(\frac{Y_t}{Y} \right)^{(1-\rho)\phi_Y}, 1 \right\}. \tag{3.51}$$

In equation (3.51), r is the steady state real interest rate, and ρ is a parameter that measures the inertia in the Taylor rule. The central bank's real asset purchases are given by

$$B_t^{CB} = \left(B^{CB}\right)^{1-\rho_B} \left(B_{t-1}^{CB}\right)^{\rho_B} \left(\frac{\Pi_t}{\Pi}\right)^{-(1-\rho_B)\psi_\Pi} \left(\frac{Y_t}{Y}\right)^{-(1-\rho_B)\psi_Y}. \tag{3.52}$$

Equation (3.52) specifies that the central bank increases the amount of government bonds purchased when inflation and aggregate income fall lower than their steady-state values since the power coefficients on these variables are assumed to be negative.

The last rule is about the amount of real reserves. By assumption, the central bank purchases new assets by issuing an equivalent amount of reserves

$$M_t = q_t B_t^{CB}. \tag{3.53}$$

Equations (3.52) and (3.53) specify how new assets purchases and reserves will move in any period t. Then, the transfer of the central bank to the fiscal authority in every period t can be determined by

$$T_t^{CB} = \left(1 + r_t^B\right) q_{t-1} B_{t-1}^{CB} - \left(1 + r_t\right) M_{t-1}. \tag{3.54}$$

3.10 Fiscal Authority

The fiscal authority obtains revenues from household lump-sum taxes, labor income taxes, and taxes on the dividends paid by all types of firms. It also issues new long-term debt $q_t B_t$ and receives a transfer from the monetary authority. These revenues are used to finance an exogenous path of real government expenditures G_t and pay the interest on previous debt. The government budget constraint in real terms is, therefore

$$G_t + \left(1 + r_t^B\right) q_{t-1} B_{t-1} = q_t B_t + \mathcal{T}_t + \tau^L w_t L_t + \tau^D \mathcal{D}_t + T_t^{CB}. \tag{3.55}$$

where the real interest rate on government bonds is given by

$$1 + r_t^B = \frac{1 + \gamma q_t}{q_{t-1}} \frac{1}{\Pi_t}. \tag{3.56}$$

The fiscal rule behind the lump-sum tax follows the tradition of Leeper (1991) and specifies the lump-sum tax as a constant amount plus a varying amount depending on the difference of previous government debt-to-GDP ratio from its steady state level counterpart

$$\mathcal{T}_t = T + \phi_B \left(\frac{q_{t-1} B_{t-1}}{Y_{t-1}} - \frac{qB}{Y} \right). \tag{3.57}$$

Given the above law of motion for lump-sum taxes, everything on the right-hand side of (3.47) is determined either exogenously, or by optimality conditions of other agents, or by policy rules. So, new debt issuance is the variable that adjusts.

3.11 Market Clearing & Equilibrium

There are several market clearing conditions in the model economy reflecting the various markets. They are all summarized by the following equations:

$$L_t = \int L_{jt} dj = \int l_{it} di \tag{3.58}$$

$$K_t = K_t^{MF} \tag{3.59}$$

$$B_t = B_t^{MF} + B_t^{CB} \tag{3.60}$$

$$Y_t = C_t + \int \Xi\left(a_{it}, a_{it-1}\right) d\Gamma_t + \xi_N N_{t-1} + I_t + \Xi_t^K\left(I_t, I_{t-1}\right) + G_t + \Xi_t^p\left(\Pi_t, \Pi_{t-1}, Y_t\right) + \xi_D D_{t-1} + \xi_A A_{t-1} \tag{3.61}$$

The set of equations (3.58) describe the labor market clearing conditions, which say that the total amount of labor supplied by households and the labor union should be equal to the total demand for labor from the side of the intermediate goods producers. Equation (3.59) is the market clearing conditions for capital claims issued by intermediate goods firms. The claims held by financial intermediaries must equal the total amount of shares issued by the firms. Equation (3.60) is the market clearing condition for government debt. The amount of bonds supplied by the government must equal the amount of bonds demanded by the private banks and the central bank. Finally, (3.61) is the goods market clearing condition which says that the total supply of goods must be equal to the goods demanded for private consumption by the households, the goods demanded for investment purposes, the goods absorbed by investment adjustment costs, the goods demanded by the government, the goods absorbed by price adjustment costs, the goods absorbed by portfolio adjustment costs, and the goods absorbed in the process of financial intermediation and in management costs.

Definition: The monetary competitive equilibrium is given by a sequence of government spending shocks $\{u_t^G\}_{t=0}^{\infty}$, policy sequences for the fiscal and monetary authorities: $\{B_t, \mathcal{T}_t, i_t^M, M_t, B_t^{CB}, T_t^{CB}\}_{t=0}^{\infty}$, value functions for households $\{V_{it}\}_{t=0}^{\infty}$ with policies $\{c_{it}, l_{it}, a_{it}, d_{it}\}_{t=0}^{\infty}$, value functions for the labor union $\{V_t^w\}_{t=0}^{\infty}$ with labor choices $\{L_t\}_{t=0}^{\infty}$, value functions for intermediate goods producers $\left\{V_t^j\right\}_{t=0}^{\infty}$ with optimal choices

$\left\{P_t^j, K_{t-1}^j, L_t^j\right\}_{t=0}^{\infty}$, value functions for capital producers $\left\{V_t^K\right\}_{t=0}^{\infty}$, with optimal choices $\{I_t, K_t\}_{t=0}^{\infty}$, prices $\{w_t, r_t^K, P_t, q_t, Q_t\}_{t=0}^{\infty}$, bank value functions $\left\{V_{nt}^{MF}\right\}_{t=0}^{\infty}$, and $\{V_{bt}\}_{t=0}^{\infty}$, with bank choices $\left\{K_{nt}^{MF}, B_{nt}^{MF}, F_{nt}^{MF}\right\}_{t=0}^{\infty}$, and $\{M_{bt}, D_{bt}\}_{t=0}^{\infty}$, domestic bank net worth $\{N_{bt}\}_{t=0}^{\infty}$ and $\left\{N_t^{MF}\right\}_{t=0}^{\infty}$, interest rates $\left\{r_t, r_t^A, r_t^D, r_t^B, r_t^F\right\}_{t=0}^{\infty}$, and a joint distribution of assets and shocks $\Gamma_t(a, d, e)$ such that in any period t:

1. Given the prices, the interest rates, and taxes the value function V_{it} satisfies the Bellman equation for household i with policies c_{it}, a_{it}, d_{it}.
2. The value function V_t^w solves the problem of the union and nominal wages are optimally set.
3. Final good firms maximize profits taking as given the prices P_t^j, P_t.
4. Intermediate goods producers maximize profits taking as given the prices w_t and r_t^K.
5. Commercial banks maximize their net worth with the optimal value being V_{bt}.
6. Investment banks choose optimally their asset holdings in order to maximize net worth V_{nt}^{MF}.
7. The investment fund satisfies its zero profit condition (3.21).
8. The capital producers maximize profits with the optimal value being V_t^K.
9. The monetary authority follows the rules (3.51)-(3.54).
10. The fiscal authority satisfies its budget constraint (3.55) and the fiscal rule (3.56).
11. The aggregate law of motion Γ is generated by the choices a_{it}, d_{it} and the matrix $\Omega(.)$.
12. The market clearing conditions (3.58)-(3.61) are satisfied.

3.12 Fiscal Stimulus with QE: The Effects on Aggregate Consumption and Investment

In general, implementing a fiscal expansion affects real wages, interest rates, and lump-sum taxes. These changes naturally lead households to change their labor supply, savings, and consumption decisions. The question arising at this point is how QE measures implemented by central banks affect the previous decisions. To understand these effects better, it is helpful to decompose the aggregate consumption function and examine the effects of various variables on it. It is well-known from the classical consumer's problem that every agent's consumption will be a function of prices, income, and taxes in equilibrium. Here, asset returns have the role of prices. Hence, since aggregate consumption is just the sum of individual consumption functions, it will be a function of the same variables, so that

$$C_t = \mathcal{C}_t\left\{\left(1-\tau^L\right) w_s L_s,\, T_s,\, r_s^D,\, r_s^A\right\}_{s\geq 0}. \tag{3.62}$$

Let $Z_s \equiv \left(1-\tau^L\right) w_s L_s$. Then, any change in aggregate consumption can be decomposed into changes in the variables of the right-hand side of (3.57) as follows

$$dC_t = \underbrace{\sum_s \frac{\partial \mathcal{C}_t}{\partial Z_s} dZ_s}_{\text{Labor Income Channel}} + \underbrace{\sum_s \frac{\partial \mathcal{C}_t}{\partial T_s} dT_s}_{\text{Lump-sum Tax Channel}} + \underbrace{\sum_s \frac{\partial \mathcal{C}_t}{\partial r_s^D} dr_s^D}_{\text{Liquid Savings Channel}} + \underbrace{\sum_s \frac{\partial \mathcal{C}_t}{\partial r_s^A} dr_s^A}_{\text{Illiquid Savings Channel}} \tag{3.63}$$

Any change in fiscal policy will induce changes in the above variables, which will be translated into changes in aggregate consumption through the labor income channel, the lump-sum tax channel, the liquid savings channel, and the illiquid savings channel. We can think of the same channels when evaluating the QE decisions of the central bank. Since, by assumption, the economy is at the ZLB and asset purchases are countercyclical, the central bank balance sheet will be reduced at the time of the fiscal expansion. This will trigger an extra change in asset prices which will change the real returns paid on these assets at the time of the shock. So, the immediate effects of asset purchases will work through the illiquid savings channel.

The previous effects on illiquid savings are only some of the effects caused by asset purchases. The change on long-term asset returns affect the resources earned by the central bank, changing the transfer made to the fiscal authority, which in turn affects new debt issuance and the lump-sum taxes imposed on households. The effects of QE on the profitability of firms and banks will also affect the lump-sum taxes. Moreover, the effect of QE on inflation will affect the real interest rate and the liquid return after the fiscal expansion. Then, all the previous effects will also lead to changes in the real wage, the consumption level, and the labor supplied by households.

On the investment side, QE will affect the real interest rate, which will change the return earned on capital, the marginal value of investment, as well as the discounting of future adjustment costs, offering stronger or weaker incentives for capital accumulation to capital goods producers.

3.13 Fiscal Multipliers

In the PHANK model, the fiscal multiplier discussed was the impact multiplier due to the nature of this linearized model. However, the nonlinear HANK model allows examining the effects of a change in fiscal policy across different horizons by using the present value multiplier $\mathcal{M}_T$ for T periods in the future

$$\mathcal{M}_T = \frac{\sum_{t=0}^{T} \frac{\Delta Y_t}{(1+r)^t}}{\sum_{t=0}^{T} \frac{\Delta G_t}{(1+r)^t}}, \tag{3.64}$$

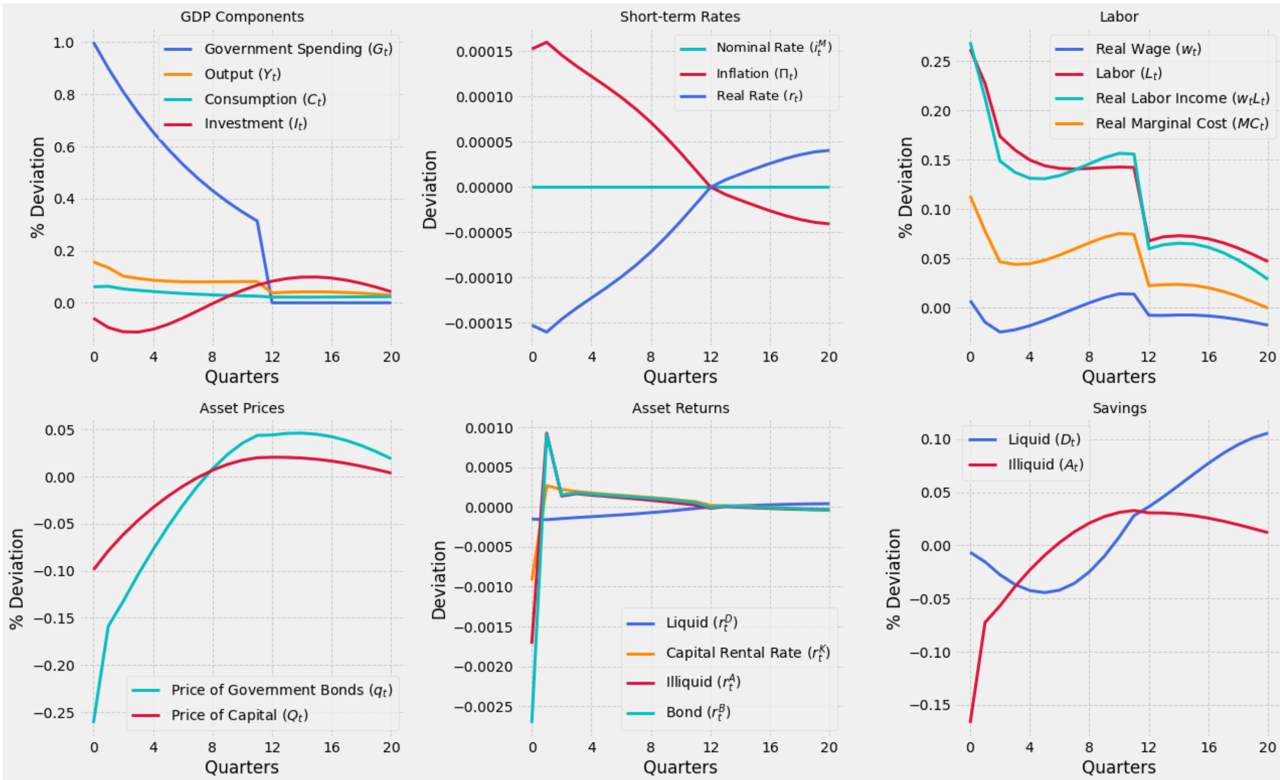
Table 3: Fiscal Multipliers & Consumption Decomposition

Fiscal Multiplier	**Zero Lower Bound**		**Taylor Rule & No QE**
	Countercyclical QE	**No QE**	
Output - Y			
$\mathcal{M}_0$	1.041	1.068	0.791
$\mathcal{M}_{12}$	1.061	1.087	1.778
Consumption - C			
$\mathcal{M}_0^C$	0.187	0.184	0.053
$\mathcal{M}_{12}^C$	0.207	0.211	0.099
Investment - I			
$\mathcal{M}_0^I$	-0.084	-0.062	-0.184
$\mathcal{M}_{12}^I$	-0.102	-0.091	0.836
Consumption Channel	**% of Total Effect on C_0**		
Labor Income	89.26	90.04	-144.56
Lump-Sum Tax	-1.59	-1.84	123.35
Liquid Savings	30.20	23.61	152.77
Illiquid Savings	-17.87	-11.81	-31.56

where r is the steady state real interest rate. The present value multiplier measures the cumulative change in aggregate output in present value terms when there is a change in government spending that starts in period 0 and ends in period T. At $t = 0$, the present value multiplier coincides with the multiplier on impact. Of course, a similar formula can also be used to compute the present value multiplier on other variables. Table 3 provides the present value fiscal multipliers for output, consumption, and investment, as well as the decomposition of the consumption response in period $t = 0$.

Government Spending Shock: In the baseline scenario, it is assumed that government spending increases in period $t = 0$ by 1% relative to its steady state value, and then this increase declines at a rate $\rho_G = 0.9$ for the next 11 quarters. At $t = 12$, that is, in the 13th quarter, government spending reverts to its steady state value. The panels of Figure 10 summarize the movements in aggregate variables and the movements in asset prices and asset returns.

Figure 10: Aggregate Responses to 1% Government Spending Shock

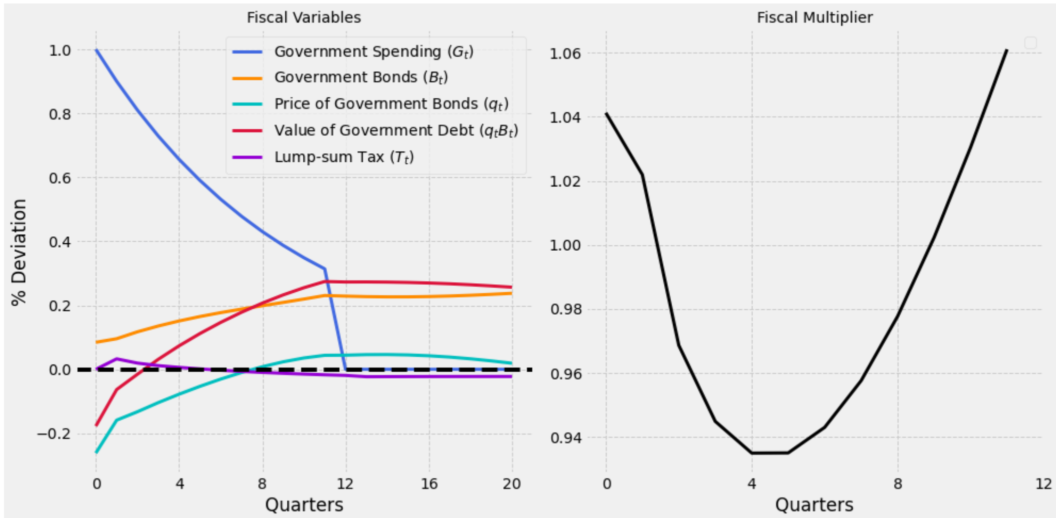


Notes: The top left panel shows the responses of various macro variables and asset prices in deviations from their steady state values, to a 1% expansionary government spending shock at the ZLB.

A deficit-financed increase in government spending by 1% at the ZLB leads to an increase in output by almost 0.156%, implying a fiscal multiplier on impact equal to $\mathcal{M}_0 = 1.041$. This number is higher than 1, in line with previous studies' findings that fiscal policy is more effective at the ZLB, although very close to 1. The increase in aggregate income initially occurs due to the higher aggregate demand implied by higher government spending. Firms respond by increasing labor demand in order to increase production. However, higher government spending is only one of the drivers of the increase in total output. Aggregate consumption also increases.

Specifically, aggregate consumption increases by around 0.061% on impact, resulting in a consumption fiscal multiplier of $\mathcal{M}_0^C = 0.187$. One reason behind this movement is that lump-sum taxes react weakly to the increase in government debt. This creates expectations for the permanent income type of households that the present discounted value of their lifetime wealth is higher, which in turn offers an incentive for higher consumption. However, given the economy's production possibilities, prices and interest rates change, giving rise to different channels that affect consumption. The two channels are related to the movements in the real returns earned by households.

Figure 11: Fiscal Responses to 1% Government Spending Shock



Notes: The left panel shows the responses of various fiscal variables in deviations from their steady state values, to a 1% expansionary government spending shock at the ZLB.

First is the liquid return channel. Due to higher aggregate demand, inflation initially increases, so the real interest rate and the real liquid return earned on bank deposits fall. The substitution effect dominates on the aggregate level in this case since the decrease in the liquid return leads to an increase in aggregate consumption at $t = 0$. This effect is responsible for around 30.20% of the total effect in aggregate consumption, which is a considerable share. Over time the liquid return rises as inflation falls, and the consumption effect becomes negative as shown in Figure 13, implying that the substitution effect still dominates in this case.

The other channel is the illiquid return channel. The illiquid return depends on the aggregate profits in the economy and the return earned from deposits in the investment bank. The aggregate profits initially fall in response to the government spending shock since the marginal cost of the intermediate goods producers increases, and the aggregate profits are heavily impacted by the profits of the intermediate goods producers. The return on the investment bank deposits falls initially because of the lower rental rate on capital which falls because of the lower price on capital. As a result, the illiquid return is initially lower. The initial decrease in the illiquid return pushes aggregate consumption lower at $t = 0$ as implied by Figure 13, implying that the income effect dominates in this case. The illiquid savings channel accounts for -17.87% of the total effect in aggregate consumption at $t = 0$, which is a considerable share.

Aggregate consumption increases after the shock in government spending also because of constrained households with *MPCs* who increase their consumption after the rise in the real labor income. The real labor income effect is the most significant since it accounts for 89.26% of the total effect on aggregate consumption.

On the other hand, investment falls on impact by around 0.059%, resulting in a fiscal multiplier for investment equal to $\mathcal{M}_0^I = -0.084$. The higher inflation rate at the time of the shock and the lower real interest rate imply higher future adjustment costs for capital producers, which discourage new investment. Due to adjustment costs, investment moves gradually. The negative effect on investment is maximized at $t = 3$ where investment decreases by 0.112%. After this quarter, investment decreases at a lower rate, and gradually increases. The effect becomes positive before investment returns to its steady state value.

As for savings, as shown in Figure 10, both liquid savings and illiquid savings initially decrease because of the lower returns earned on both types of savings. Initially inflation lowers the real liquid return and lower profits lower the real illiquid return. Illiquid savings increase over time as the illiquid rate rises over time due to the higher rate paid by the investment bank and the profits reverting to steady state.

Figure 11 contains the movements of the fiscal variables. The left panel shows that lump-sum taxes increase, but the responce to the fiscal changes is weak since by assumption $\phi_B = 0.001$. The amount of government bonds issued increases due to the higher spending. The price of the bonds follows a path that depends negatively on the expected capital rental rates and investment bank interest rates from equation (3.29). These two returns increase after the shock at $t = 1$, and the bond price at $t = 0$ falls. As these returns fall over time, the price of the bonds increases and reverts to its steady state value. The value of the debt follows the path of the bond price: it initially falls and then increases but does not explode due to the low interest payments. The right panel of Figure 11 depicts the present value fiscal multiplier for the 12 periods the shock lasts. We see that the fiscal multiplier takes its maximum value at $t = 12$.

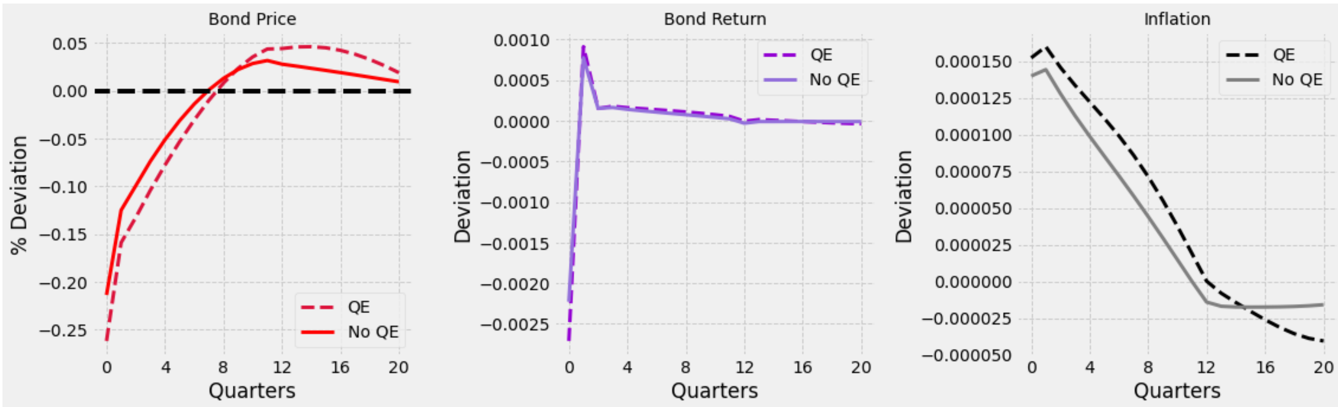
3.14 The Role of Quantitative Easing

The central question of this paper is about explaining the channels through which quantitative easing affects the fiscal multiplier and how strong these effects are. To answer these questions, the same fiscal experiment as in the previous subsection is considered, but now the central bank does not adopt unconventional monetary measures, so $\psi_\Pi = \psi_Y = 0$. To understand how QE affects inflation in a regime where fiscal policy is active and monetary policy is passive consider the present value government budget constraint.

$$\frac{(1+\gamma q_t)\,B_{t-1}}{\Pi_t} = \sum_{j=0}^{\infty}\left(\prod_{k=1}^{j|_{j\geq 1}} \frac{1}{1+r^B_{t+k}}\right)\left(\mathcal{T}_{t+j} + \tau^L w_{t+j} L_{t+j} + \tau^D \mathcal{D}_{t+j} + T^{CB}_{t+j} - G_{t+j}\right) \tag{3.65}$$

Equation (3.65) is an asset pricing equation, as noted in Cochrane (2022), stating that legacy debt is a claim to the present value of future primary surpluses. Any change in future surpluses will make inflation and the price of the bonds at time t adjust so that the equality in (3.65) holds. The central bank's actions mainly affect the paths of asset prices and asset returns. These effects, in turn, determine the movements of inflation.

Figure 12: Inflation Determinants Responses to 1% Government Spending Shock with and without QE



Notes: The left panel shows the response of the government bond price under the scenario of QE and no QE. The middle panel shows the responses of government bond yields, while the right panel shows the responses of inflation.

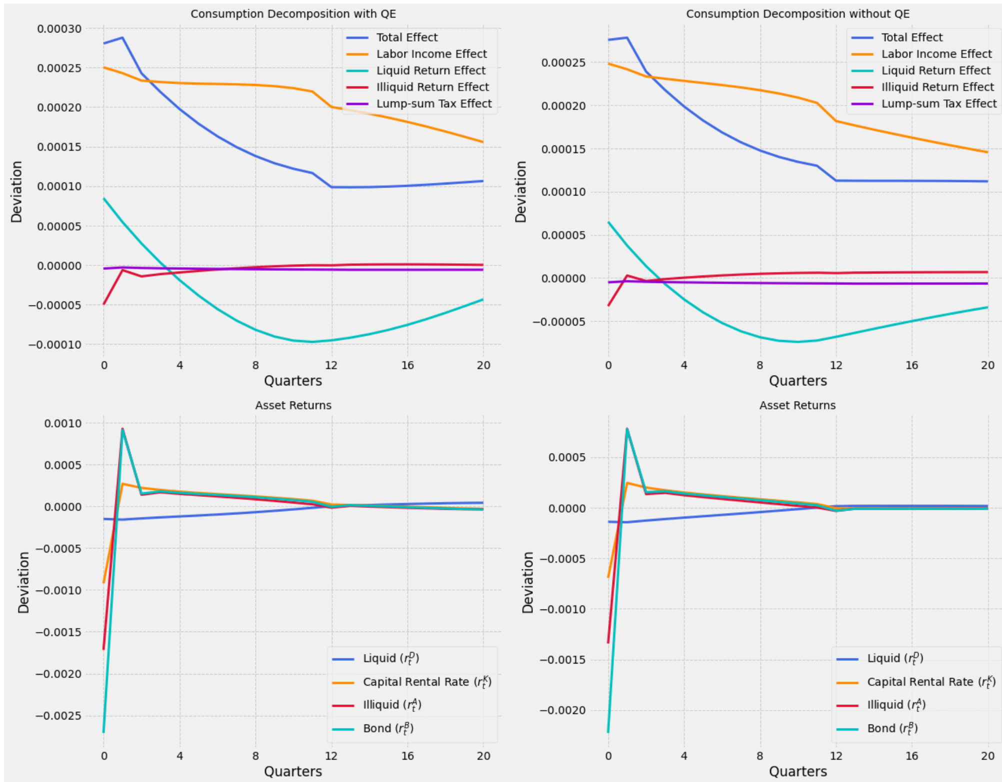
When countercyclical QE is implemented, future returns on government bonds are expected to be higher over time, so inflation increases. The logic is that higher interest rates in the future create wealth effects for bond holders that lead to higher aggregate demand and inflation since fiscal policy is not running surpluses that would force households to save and bring aggregate demand lower. Figure 12 confirms the previous.

Government Spending Shock: We now consider a 12-quarter shock to government purchases as in the previous subsection. As expected, since there is no countercyclical QE, the fiscal multiplier on impact is higher and equal to $\mathcal{M}_0 = 1.068$, being higher than the impact multiplier with QE by 2.59%. The present value multipliers remain higher over the 12 quarters. In the last period, the present value multiplier is $\mathcal{M}_{12} = 1.087$, which is almost 2.45% higher than the case with unconventional monetary policy.

Figure 13 shows that QE works through various channels affecting the households. The liquid savings channel without QE is in general weaker than the case of QE, possibly due to lower inflation without QE which makes the real liquid return slightly higher and triggers the substitution effects. The real labor income channel follows a path very similar to the case of QE. The real illiquid return is higher since at $t = 0$ the rental rate on capital is higher without QE, due to higher marginal costs, which increases the rate that investment banks pay to the investment fund, and the rate that the investment fund pays to households.

The effect of QE on investment results from the lower inflation without countercyclical QE which results in higher real interest rates. The higher real rates imply heavier discounting on future adjustment costs from the side of the capital producers. This provides incentives for higher capital accumulation. The investment multiplier on impact is $\mathcal{M}^I_0 = -0.062$, and the present value multiplier is $\mathcal{M}^I_{12} = -0.091$, which are both lower in absolute value than the case with QE.

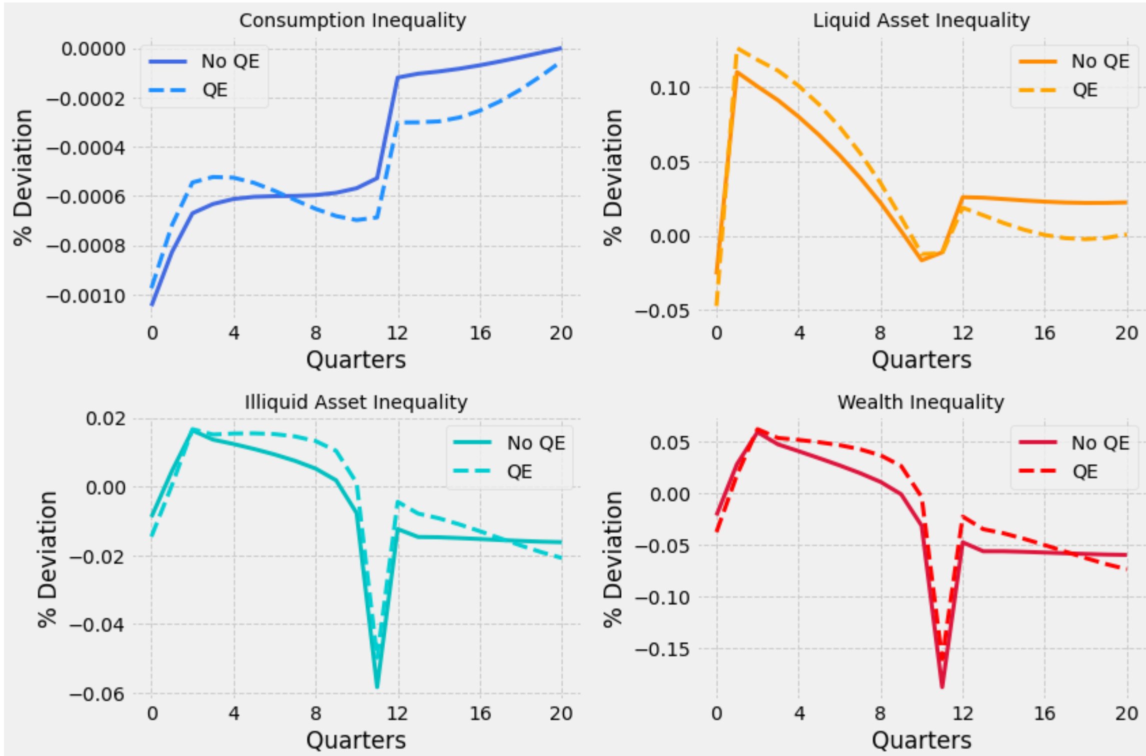
Figure 13: Consumption Decomposition to 1% Government Spending Shock with and without QE



Notes: The above panels show the decomposition of the response of aggregate consumption with QE (left panels) and without QE (right panels) to a 1% expansionary government spending shock at the ZLB.

The effects of QE on inequality are shown in Figure 14, where the mean log deviation of liquid asset holdings, illiquid asset holdings, household wealth, and consumption, is plotted under the scenario of countercyclical QE and the alternative scenario of no QE. The fiscal shock reduces consumption inequality initially and wealth inequality over time. With countercyclical QE consumption inequality is lower than the no QE case over time, but higher in the first six quarters. This is mainly due to the lower realized illiquid return which is the result of lower profits with countercyclical QE over time, but higher profits initially. This leads the richer households to reduce their consumption over time. On the other hand, the wealth inequality measures rise over time. With countercyclical QE, inflation is higher initially, as suggested by Figure 12, which disincentivizes the poorer households to hold the liquid asset, relative to the no QE case. Moreover, the poorer households cannot afford the transaction cost associated with the illiquid asset. So, when QE is countercyclical, and labor income is lower, poorer households accumulate less illiquid assets, increasing illiquid asset inequality. In addition, countercyclical QE increases expected returns, incentivizing richer households to increase their illiquid asset holdings. Total wealth inequality follows the path of illiquid asset inequality because the illiquid asset has the biggest share in total wealth.

Figure 14: Inequality Responses to 1% Government Spending Shock with and without QE



Notes: The above panels show the evolution of the variance of the log-deviation of household liquid asset holdings, illiquid asset holdings, total wealth, consumption, after an expansionary 1% shock in government consumption.

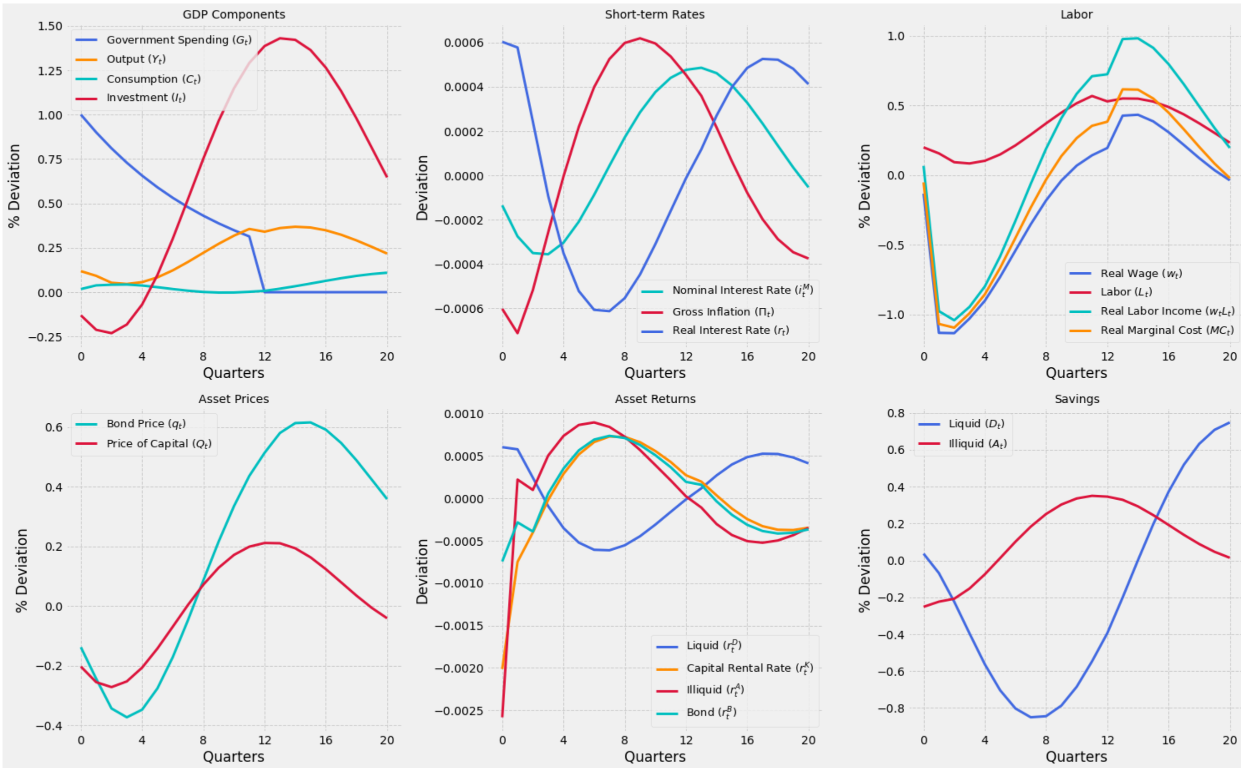
3.15 Away from the Zero Lower Bound: Taylor Rule

Now I examine the size of the fiscal multipliers outside the ZLB where the monetary authority follows a Taylor rule such as the one given in equation (3.51). The values assigned to policy parameters are $\phi_\Pi = 1.5$, $\phi_Y = 0$ for the nominal interest rate, $\psi_\Pi = 0$ and $\psi_Y = 0$ since QE is not usually used outside of the ZLB, and $\phi^B = 0.2$ for the lump-sum tax.

Government Spending Shock: Figure 15 summarizes the IRFs of aggregate variables after a positive shock in government spending. Now the countercyclical response of the nominal interest rate following the fiscal expansion leads to a significantly lower increase in output. The multiplier on impact is equal to $\mathcal{M}_0 = 0.791$, and the present value multiplier is equal to $\mathcal{M}_{12} = 1.778$.

After the fiscal expansion, the real interest rate increases, leading to slightly higher consumption but lower liquid savings. Also, the strong reaction of lump-sum taxes creates expectations in the permanent income type of households that the present discounted value of their lifetime wealth is lower this time so the consumption response is not strong. On top of that, labor income falls making the consumption response

Figure 15: Aggregate Responses to 1% Government Spending Shock under a Taylor Rule

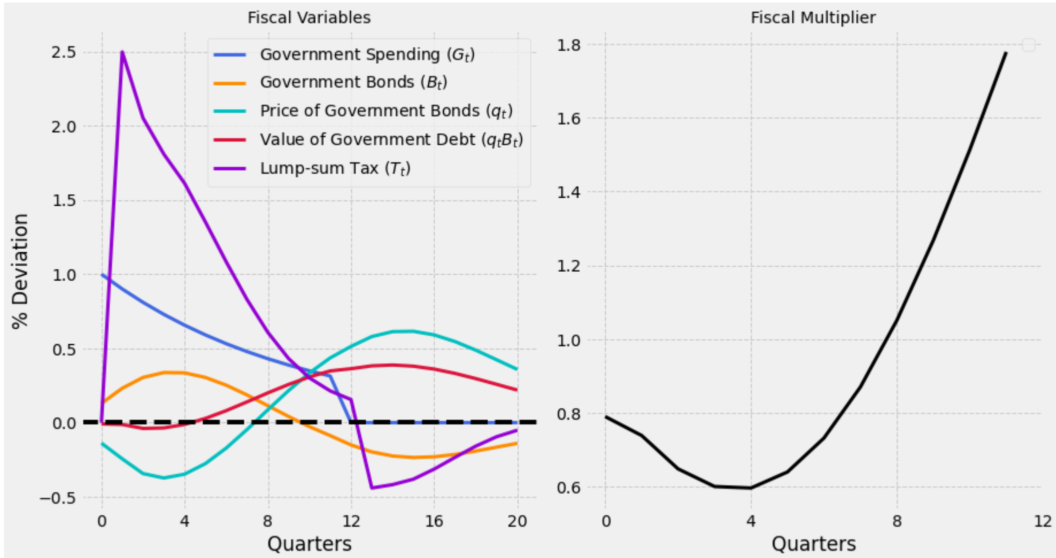


Notes: The top left panel shows the responses of various aggregate variables in deviations from their steady state values, to a 1% expansionary government spending shock outside of the ZLB, under a Taylor rule.

weaker. As the real marginal cost declines, the firms of intermediate goods producers initially increase. On the other hand, investment initially falls as aggregate demand is weaker and there is less demand for capital. This drives lower the price of capital and the realized return on capital. So initially, the investment banks experience a decrease in their net worth and invest less in capital. In addition, the price of government bonds initially falls as more debt is issued, and the realized return on government bonds at $t = 0$ falls significantly, driving lower the return that the investment banks pay to the investment fund. As the returns overshoot in the next periods, the illiquid savings also increase.

On the fiscal side, the amount of government bonds issued increases, and the price of the bonds falls symmetrically, but at a slightly higher rate, so the value of the debt falls slowly. On the other hand, lump-sum taxes react strongly to the increase in government debt and preclude an explosive debt path. The increased profits of the firm also bring in more revenues for the government. The fiscal multipliers increase over time, as seen in the right panel of Figure 16, due to the increase in investment which leads to higher output over time.

Figure 16: Fiscal Responses to 1% Government Spending Shock under a Taylor Rule



Notes: The left panel shows the responses of various fiscal variables in deviations from their steady state values, to a 1% expansionary government spending shock outside of the ZLB and under a Taylor rule.

4 Concluding Remarks

In this paper, I focused on the effects of quantitative easing policies on the fiscal multiplier and inequality. Both the models discussed showed that fiscal policy becomes more effective when quantitative easing policies are accommodative. The government spending multipliers computed at the zero lower bound exceeded 1 in both models, whereas outside of the zero lower bound, both models predict low fiscal multipliers lower than 1. In addition, countercyclical QE tends to increase wealth inequality since it reduces the positive effects of fiscal expansion, and poorer households receive lower labor income and accumulate less assets, driving wealth inequality higher. Nevertheless, countercyclical QE tends to decrease consumption inequality over time since it leads to lower profits over time and makes which makes richer households consume less.

References

A. Auclert and M. Rognlie. Inequality and aggregate demand. Technical report, National Bureau of Economic Research, 2018. 6

A. Auclert, M. Rognlie, and L. Straub. The intertemporal keynesian cross. Technical report, National Bureau of Economic Research, 2018. 6

A. Auclert, M. Rognlie, and L. Straub. Micro jumps, macro humps: Monetary policy and business cycles in an estimated hank model. Technical report, National Bureau of Economic Research, 2020. 6

A. Auclert, B. Bardóczy, and M. Rognlie. Mpcs, mpes, and multipliers: A trilemma for new keynesian models. *The Review of Economics and Statistics*, pages 1–41, 2021a. 39

A. Auclert, B. Bardóczy, M. Rognlie, and L. Straub. Using the sequence-space jacobian to solve and estimate heterogeneous-agent models. *Econometrica*, 89(5):2375–2408, 2021b. 92

F. O. Bilbiie. Monetary policy and heterogeneity: An analytical framework. *Available at SSRN 3106805*, 2018. 5

L. Christiano, M. Eichenbaum, and S. Rebelo. When is the government spending multiplier large? *Journal of Political Economy*, 119(1):78–121, 2011. 5, 19, 26, 30, 32

J. Cochrane. The fiscal theory of the price level. 2022. 55

J. H. Cochrane. The new-keynesian liquidity trap. *Journal of Monetary Economics*, 92:47–63, 2017. 5

W. Cui and V. Sterk. Quantitative easing with heterogeneous agents. *Journal of Monetary Economics*, 123:68–90, 2021. 6

V. Dimakopoulou, G. Economides, and A. Philippopoulos. The ecb's policy, the recovery fund and the importance of trust and fiscal corrections: The case of greece. *Economic Modelling*, 112:105846, 2022. 6

B. Dupor, M. Karabarbounis, M. Kudlyak, and M. Mehkari. Regional consumption responses and the aggregate fiscal multiplier. *Available at SSRN 3131248*, 2018. 5

G. B. Eggertsson. What fiscal policy is effective at zero interest rates? *NBER Macroeconomics Annual*, 25(1): 59–112, 2011. 16, 17

E. Farhi and I. Werning. Fiscal multipliers: Liquidity traps and currency unions. In *Handbook of macroeconomics*, volume 2, pages 2417–2492. Elsevier, 2016. 5

R. C. Feenstra, R. Inklaar, and M. P. Timmer. The next generation of the penn world table. *American Economic Review*, 105(10):3150–3182, 2015. 90

M. Gertler and P. Karadi. A model of unconventional monetary policy. *Journal of monetary Economics*, 58(1): 17–34, 2011. 5, 42, 89, 91

M. Gertler and P. Karadi. QE 1 vs. 2 vs. 3...: A framework for analyzing large-scale asset purchases as a monetary policy tool. *29th issue (January 2013) of the International Journal of Central Banking*, 2013. 5, 42, 91

M. Hagedorn, J. Luo, I. Manovskii, and K. Mitman. Forward guidance. *Journal of Monetary Economics*, 102: 1–23, 2019a. 6

M. Hagedorn, I. Manovskii, and K. Mitman. The fiscal multiplier. Technical report, National Bureau of Economic Research, 2019b. 5, 39

G. Kaplan, B. Moll, and G. L. Violante. Monetary policy according to hank. *American Economic Review*, 108 (3):697–743, 2018. 6

A. Kyriazis. Quantitative easing spillovers. *Available at SSRN 4499714*, 2022. 6, 89, 90, 91

D. Lee. Quantitative easing and inequality. 2020. 6, 90, 91

E. M. Leeper. Equilibria under activeand passivemonetary and fiscal policies. *Journal of monetary Economics*, 27(1):129–147, 1991. 48

A. McKay, E. Nakamura, and J. Steinsson. The power of forward guidance revisited. *American Economic Review*, 106(10):3133–58, 2016. 16, 17, 89, 91

M. O. Ravn and V. Sterk. Job uncertainty and deep recessions. *Journal of Monetary Economics*, 90:125–141, 2017. 6

J. J. Rotemberg. Monopolistic price adjustment and aggregate output. *The Review of Economic Studies*, 49(4): 517–531, 1982. 10, 39, 46

E. Sims and J. C. Wu. Evaluating central banks tool kit: Past, present, and future. *Journal of Monetary Economics*, 118:135–160, 2021. 16, 17

E. Sims, J. C. Wu, and J. Zhang. The four equation new keynesian model. *The Review of Economics and Statistics*, pages 1–45, 2021. 5, 6, 16, 17, 28

E. R. Sims, J. C. Wu, and J. Zhang. Unconventional monetary policy according to hank. *Available at SSRN*, 2022. 6

M. Woodford. Fiscal requirements for price stability. Technical report, National Bureau of Economic Research, 2001. 13

M. Woodford. Simple analytics of the government expenditure multiplier. *American Economic Journal: Macroeconomics*, 3(1):1–35, 2011. 5

A Algebraic Derivations

In this appendix I present the algebraic manipulations behind some of the equations that appear in the two models in the main text.

A.1 Three-Agent Model: Proof of Proposition 1

Proposition 1: *The dynamic IS curve has the following form*

$$\hat{y}_t = \mathbb{E}_t \hat{y}_{t+1} - \Gamma_g \left(\mathbb{E}_t \hat{g}_{t+1} - \hat{g}_t \right) - \Xi_c \left(i_t^M - \mathbb{E}_t \pi_{t+1} - \rho_c \right) - \Xi_p \left(\mathbb{E}_t \hat{m}_{t+1} - \hat{m}_t \right). \tag{A.1}$$

Proof: First I derive the aggregate equilibrium conditions of the Three-Agent Model presented in the main text. I start from the demand side. From the labor supply optimality conditions for each group we have that

$$\mu_L L_{pt}^{\nu} = C_{pt}^{-\sigma} w_t \tag{A.2}$$

$$\mu_L L_{ct}^{\nu} = C_{ct}^{-\sigma} w_t \tag{A.3}$$

$$\mu_L L_{ht}^{\nu} = C_{ht}^{-\sigma} w_t. \tag{A.4}$$

In the main text I have assumed that the transfers T_p and T_h received by the parents and the hand-to-mouth households respectively are set in such a way that $C_p = C_c = C_h = C$ at the steady state. Then, equations (A.2), (A.3) and (A.4) imply that $L_p = L_c = L_h = L$. The consumption and labor supply equality across households at the steady state is needed in order to be able to express the real wage as a function of aggregate consumption and aggregate labor, which then allows to express the consumption of the hand-to-mouth as a function of aggregate consumption and aggregate income. This in turn allows to derive the IS curve presented in the main text. Specifically, I first log-linearize the definitions of aggregate consumption and aggregate labor taking into account the previous simplifications:

$$C_t = \eta_p C_{pt} + \eta_c C_{ct} + \eta_h C_{ht} \Rightarrow C\hat{c}_t = \eta_p C_p \hat{c}_{pt} + \eta_c C_c \hat{c}_{ct} + \eta_h C_h \hat{c}_{ht} \Rightarrow \hat{c}_t = \eta_p \hat{c}_{pt} + \eta_c \hat{c}_{ct} + \eta_h \hat{c}_{ht} \tag{A.5}$$

$$L_t = \eta_p L_{pt} + \eta_c L_{ct} + \eta_h L_{ht} \Rightarrow L\hat{l}_t = \eta_p L_p \hat{l}_{pt} + \eta_c L_c \hat{l}_{ct} + \eta_h L_h \hat{l}_{ht} \Rightarrow \hat{l}_t = \eta_p \hat{l}_{pt} + \eta_c \hat{l}_{ct} + \eta_h \hat{l}_{ht}, \tag{A.6}$$

where in the previous equation any variable $\hat{x}_t$ denotes percentage deviations of X_t from its steady state value X. Next, by log-linearizing around the steady state, and then multiplying both sides of equations (A.2), (A.3) and (A.4) with the size of the corresponding households and adding by parts we get:

$$\sigma\hat{c}_t + \nu\hat{l}_t = \eta_p\sigma\hat{c}_{pt} + \eta_p\nu\hat{l}_{pt} + \eta_c\sigma\hat{c}_{ct} + \eta_c\nu\hat{l}_{ct} + \eta_h\sigma\hat{c}_{ht} + \eta_h\nu\hat{l}_{ht} = \eta_p\hat{w}_t + \eta_c\hat{w}_t + \eta_h\hat{w}_t = \hat{w}_t \tag{A.7}$$

The log-linearized production function is:

$$\hat{y}_t = \hat{l}_t. \tag{A.8}$$

The log-lizearized equilibrium conditions for the hand-to-mouth consumers are

$$\hat{c}_{ht} = \frac{wL}{C}\left(\hat{w}_t + \hat{l}_{ht}\right) \tag{A.9}$$

$$\nu\hat{l}_{ht} = -\sigma\hat{c}_{ht} + \hat{w}_t. \tag{A.10}$$

If we combine equations (A.7), (A.9) and (A.10) we can derive the consumption function of the hand-to-mouth as

$$\hat{c}_{ht} = \frac{(\nu+1)\frac{wL}{C}}{\nu + \sigma\frac{wL}{C}}\left(\sigma\hat{c}_t + \nu\hat{y}_t\right). \tag{A.11}$$

Turn now to the market clearing condition in the goods market:

$$\hat{y}_t = (1-g)\,\hat{c}_t + g\hat{g}_t \tag{A.12}$$

Consider next the log-linearized budget constraint and the labor supply condition for the long-term savers

$$\hat{c}_{pt} = \frac{wL}{C}\left(\hat{w}_t + \hat{l}_{pt}\right) + \frac{\mathcal{D}^{IG}}{C}\hat{d}_t^{IG} - \frac{qb_p}{C}\left(\hat{q}_t + \hat{b}_{pt}\right) \tag{A.13}$$

$$\nu\hat{l}_{pt} = -\sigma\hat{c}_{pt} + \hat{w}_t. \tag{A.14}$$

The log-linear expression for the profits of intermediate goods producers is the following

$$\hat{d}_t^{IG} = \frac{Y}{\mathcal{D}^{IG}}\hat{y}_t - \frac{wL}{\mathcal{D}^{IG}}\left(\hat{w}_t + \hat{y}_t\right). \tag{A.15}$$

The market clearing condition in the market for government debt is

$$qb = \eta_p q_t b_{pt} + q_t b_{bt} + q_t b_t^{CB} \Rightarrow 0 = \left(\eta_p b_p + b_b + b^{CB}\right) \hat{q}_t + \eta_p b_p \hat{b}_{pt} + b_b \hat{b}_{bt} + b^{CB} \hat{b}_t^{CB} \Rightarrow$$
$$\hat{b}_{pt} = -\frac{1}{\eta_p b_p} \left[\left(\eta_p b_p + b_b + b^{CB}\right) \hat{q}_t + b_b \hat{b}_{bt} + b^{CB} \hat{b}_t^{CB} \right]. \tag{A.16}$$

In equation (A.16) any variable with a hat refers to the log-deviation of the corresponding real bonds from their steady state value. The log-linearized leverage constraint of private banks and the log-linearized QE equilibrium condition are:

$$\hat{q}_t + \hat{b}_{bt} = 0 \tag{A.17}$$
$$\hat{q}_t + \hat{b}_t^{CB} = \hat{m}_t. \tag{A.18}$$

If we now combine equations (A.16), (A.17) and (A.18) we can solve for $\hat{b}_{pt}$ as

$$\hat{b}_{pt} = -\hat{q}_t - \frac{b^{CB}}{\eta_p b_p} \hat{m}_t. \tag{A.19}$$

Then, equations (A.12)-(A.16) and (A.19) imply that the consumption function of the long-term savers is

$$\hat{c}_{pt} = \frac{\frac{wL}{C}\frac{\nu+1}{\nu}\left(\frac{\sigma}{1-g}+\nu\right) + \frac{Y}{C}\left[1 - w\left(\frac{\sigma}{1-g}+\nu+1\right)\right]}{1+\frac{\sigma}{\nu}\frac{wL}{C}} \hat{y}_t - \frac{\frac{1}{\nu}\frac{wL}{C}\frac{\sigma g}{1-g}}{1+\frac{\sigma}{\nu}\frac{wL}{C}} \hat{g}_t + \frac{qB^{CB}}{\eta_p PC}\frac{1}{1+\frac{\sigma}{\nu}\frac{wL}{C}} \hat{m}_t. \tag{A.20}$$

From the Euler equation for parents we have that

$$\mathbb{E}_t \left(i_t^B - \pi_{t+1} - \rho_p \right) = \sigma \left(\mathbb{E}_t \hat{c}_{pt+1} - \hat{c}_{pt} \right), \tag{A.21}$$

where the difference between consumption in periods t and $t+1$ can be derived from equation (A.20). Next, the loglinearized Euler equations for short-term savers and long-term savers, weighted by their respective sizes are

$$\eta_c \hat{c}_{ct} = \eta_c \mathbb{E}_t \hat{c}_{ct+1} - \frac{\eta_c}{\sigma} \left(i_t^M - \mathbb{E}_t \pi_{t+1} - \rho_c \right) \tag{A.22}$$
$$\eta_p \hat{c}_{pt} = \eta_p \mathbb{E}_t \hat{c}_{pt+1} - \frac{\eta_p}{\sigma} \mathbb{E}_t \left(i_t^B - \pi_{t+1} - \rho_p \right). \tag{A.23}$$

where $i_t^M = i_t^D$ by the private bank's optimality condition. Next, we can add by parts equations (A.22) and (A.23) and get

$$
\begin{aligned}
\eta_c \hat{c}_{ct} + \eta_p \hat{c}_{pt} &= \eta_c \mathbb{E}_t \hat{c}_{ct+1} - \frac{\eta_c}{\sigma}\left(i_t^M - \mathbb{E}_t \pi_{t+1} - \rho_c\right) + \eta_p \mathbb{E}_t \hat{c}_{pt+1} - \frac{\eta_p}{\sigma}\mathbb{E}_t\left(i_t^B - \pi_{t+1} - \rho_p\right) \overset{(A.5)}{\Longrightarrow} \\
\hat{c}_t - \eta_h \hat{c}_{ht} &= \mathbb{E}_t \hat{c}_{t+1} - \eta_h \mathbb{E}_t \hat{c}_{ht+1} - \frac{\eta_c}{\sigma}\left(i_t^M - \mathbb{E}_t \pi_{t+1} - \rho_c\right) - \frac{\eta_p}{\sigma}\mathbb{E}_t\left(i_t^B - \pi_{t+1} - \rho_p\right) \overset{(A.11),(A.12),(A.20),(A.21)}{\Longrightarrow} \\
\hat{y}_t &= \mathbb{E}_t \hat{y}_{t+1} - \Gamma_g\left(\mathbb{E}_t \hat{g}_{t+1} - \hat{g}_t\right) - \Xi_c\left(i_t^M - \mathbb{E}_t \pi_{t+1} - \rho_p\right) - \Xi_p\left(\mathbb{E}_t \hat{m}_{t+1} - \hat{m}_t\right). \qquad \text{(A.1)}
\end{aligned}
$$

with

$$
\Gamma_g \equiv \frac{\frac{g}{1-g} - \eta_h \frac{(\nu+1)\frac{wL}{C}}{\nu + \sigma\frac{wL}{C}} \frac{\sigma g}{1-g} - \frac{\frac{\eta_p}{\nu}\frac{wL}{C}\frac{\sigma g}{1-g}}{1+\frac{\sigma}{\nu}\frac{wL}{C}}}{\frac{1}{1-g} - \eta_h \frac{(\nu+1)\frac{wL}{C}}{\nu+\sigma\frac{wL}{C}}\left[\frac{\sigma}{1-g}+\nu\right] - \eta_p \frac{\frac{wL}{C}\frac{\nu+1}{\nu}\left(\frac{\sigma}{1-g}+\nu\right)+\frac{Y}{C}\left[1-w\left(\frac{\sigma}{1-g}+\nu+1\right)\right]}{1+\frac{\sigma}{\nu}\frac{wL}{C}}}
$$

$$
\Xi_c \equiv \frac{\eta_c}{\sigma} \frac{1}{\frac{1}{1-g} - \eta_h \frac{(\nu+1)\frac{wL}{C}}{\nu+\sigma\frac{wL}{C}}\left[\frac{\sigma}{1-g}+\nu\right] - \eta_p \frac{\frac{wL}{C}\frac{\nu+1}{\nu}\left(\frac{\sigma}{1-g}+\nu\right)+\frac{Y}{C}\left[1-w\left(\frac{\sigma}{1-g}+\nu+1\right)\right]}{1+\frac{\sigma}{\nu}\frac{wL}{C}}}
$$

$$
\Xi_p \equiv \frac{qB^{CB}}{PC\left(1+\frac{\sigma}{\nu}\frac{wL}{C}\right)} \frac{1}{\frac{1}{1-g} - \eta_h \frac{(\nu+1)\frac{wL}{C}}{\nu+\sigma\frac{wL}{C}}\left[\frac{\sigma}{1-g}+\nu\right] - \eta_p \frac{\frac{wL}{C}\frac{\nu+1}{\nu}\left(\frac{\sigma}{1-g}+\nu\right)+\frac{Y}{C}\left[1-w\left(\frac{\sigma}{1-g}+\nu+1\right)\right]}{1+\frac{\sigma}{\nu}\frac{wL}{C}}}
$$

As regards the New Keynesian Phillips Curve given by equation (2.43) this is derived in the usual way by log-linearizing equation (2.20) around the steady state.

A.2 Three-Agent Model: Proof of Proposition 2

Proposition 2: *If government spending is the only state variable, then the fiscal multiplier on impact is given by the following expression:*

$$\mathcal{M}_0 = \frac{1}{g}\frac{\Gamma_g\left(1-\rho_G\right)+\left[\Xi_c\left(\phi_\Pi-\rho_G\right)+\Xi_p\psi_\Pi\left(1-\rho_G\right)\right]\frac{\kappa}{\left(1-\beta_p\rho_G\right)}\frac{\sigma g}{1-g}}{\left(1-\rho_G\right)\left(1+\Xi_p\psi_Y\right)+\Xi_c\phi_Y+\left[\Xi_c\left(\phi_\Pi-\rho_G\right)+\Xi_p\psi_\Pi\left(1-\rho_G\right)\right]\frac{\kappa}{1-\beta_p\rho_G}\left(\frac{\sigma}{1-g}+\nu\right)} \tag{A.24}$$

Proof: First we guess that the solutions for $\hat{y}_t$ and π_t in equations (2.42) and (2.43) will have a linear form

$$\hat{y}_t = A_y\hat{g}_t \tag{A.25}$$

$$\pi_t = A_\pi\hat{g}_t. \tag{A.26}$$

The coefficients A_y and A_π can be determined by combining equations (2.42)-(2.46) with (A.25) and (A.26)

$$A_y = \frac{\Gamma_g\left(1-\rho_G\right)+\left[\Xi_c\left(\phi_\Pi-\rho_G\right)+\Xi_p\psi_\Pi\left(1-\rho_G\right)\right]\frac{\kappa}{\left(1-\beta_p\rho_G\right)}\frac{\sigma g}{1-g}}{\left(1-\rho_G\right)\left(1+\Xi_p\psi_Y\right)+\Xi_c\phi_Y+\left[\Xi_c\left(\phi_\Pi-\rho_G\right)+\Xi_p\psi_\Pi\left(1-\rho_G\right)\right]\frac{\kappa}{1-\beta_p\rho_G}\left(\frac{\sigma}{1-g}+\nu\right)}$$

$$A_\pi = \frac{\kappa}{1-\beta_p\rho_G}\left[\left(\frac{\sigma}{1-g}+\nu\right)A_y-\frac{\sigma g}{1-g}\right].$$

However, since the model is log-linearized around the steady state, $A_y = \frac{\%dY_0}{\%dG_0} = \frac{dY_0}{dG_0}\frac{G_0}{Y_0}$ is the elasticity of output with respect to government spending. The fiscal multiplier on impact is the corresponding derivative $\frac{dY_0}{dG_0}$, so we need to adjust for the term $\frac{G_0}{Y_0}$. Since by assumption the economy starts from the steady state, so that $Y_0 = Y$ and $G_0 = G$, dividing A_y by $g = \frac{G}{Y}$ gives the expression in (2.48) in the main text.

$$\mathcal{M}_0 = \frac{dY_0}{dG_0} = \frac{A_y}{g}.$$

A.3 Three-Agent Model: Proof of Proposition 3

Proposition 3: *If government spending and real reserves are the state variables, then the fiscal and quantitative easing multipliers satisfy*

$$\mathcal{M}_0^G = \frac{1}{g}\frac{\Gamma_g\left(1-\rho_G\right)+\Xi_c\left(\phi_\Pi-\rho_G\right)\dfrac{\kappa}{\left(1-\beta_p\rho_G\right)}\dfrac{\sigma g}{1-g}}{1-\rho_G+\Xi_c\phi_Y+\Xi_c\left(\phi_\Pi-\rho_G\right)\dfrac{\kappa}{1-\beta_p\rho_G}\left(\dfrac{\sigma}{1-g}+\nu\right)} \tag{A.27}$$

$$\mathcal{M}_0^M = \frac{1}{m}\frac{\Xi_p\left(1-\rho_M\right)}{1-\rho_M+\Xi_c\phi_Y+\Xi_c\left(\phi_\Pi-\rho_M\right)\dfrac{\kappa}{1-\beta_p\rho_M}\left(\dfrac{\sigma}{1-g}+\nu\right)}. \tag{A.28}$$

Proof: First we guess that the solutions for $\hat{y}_t$ and π_t in equations (2.42) and (2.43) will have a linear form

$$\hat{y}_t = A_y^G\hat{g}_t + A_y^M\hat{m}_t \tag{A.29}$$

$$\pi_t = A_\pi^G\hat{g}_t + A_\pi^M\hat{m}_t. \tag{A.30}$$

The coefficients A_y^G, A_y^M, A_π^G and A_π^M can be determined by combining equations (2.42), (2.43), (2.44), (2.46), (2.48) with (A.29) and (A.30)

$$A_y^G = \frac{\Gamma_g\left(1-\rho_G\right)+\Xi_c\left(\phi_\Pi-\rho_G\right)\dfrac{\kappa}{\left(1-\beta_p\rho_G\right)}\dfrac{\sigma g}{1-g}}{1-\rho_G+\Xi_c\phi_Y+\Xi_c\left(\phi_\Pi-\rho_G\right)\dfrac{\kappa}{1-\beta_p\rho_G}\left(\dfrac{\sigma}{1-g}+\nu\right)}$$

$$A_\pi^G = \frac{\kappa}{1-\beta_p\rho_G}\left[\left(\frac{\sigma}{1-g}+\nu\right)A_y^G-\frac{\sigma g}{1-g}\right]$$

$$A_y^M = \frac{\Xi_p\left(1-\rho_M\right)}{1-\rho_M+\Xi_c\phi_Y+\Xi_c\left(\phi_\Pi-\rho_M\right)\dfrac{\kappa}{1-\beta_p\rho_M}\left(\dfrac{\sigma}{1-g}+\nu\right)}$$

$$A_\pi^M = \frac{\kappa}{1-\beta\rho_M}\left(\frac{\sigma}{1-g}+\nu\right)A_y^M.$$

Then, the fiscal and the quantitative easing multipliers are given by taking the partial elasticities of output with respect to government spending and real reserves and adjusting by their initial steady state values relative to steady state output

$$\mathcal{M}_0^G = \frac{A_y^G}{g}$$
$$\mathcal{M}_0^M = \frac{A_y^M}{m}.$$

A.4 Three-Agent Model: Proof of Proposition 4

Proposition 4: *If government spending is the only state variable, the fiscal multiplier at the zero lower bound is given by the following expression*

$$\mathcal{M}_0^{ZLB} = \frac{1}{g}\frac{\Gamma_g\left(1-\delta\right)+\left[\Xi_p\psi_\Pi\left(1-\delta\right)-\delta\Xi_c\right]\frac{\kappa}{\left(1-\beta_p\delta\right)}\frac{\sigma g}{1-g}}{\left(1-\delta\right)\left(1+\Xi_p\psi_Y\right)+\left[\Xi_p\psi_\Pi\left(1-\delta\right)-\delta\Xi_c\right]\frac{\kappa}{1-\beta_p\delta}\left(\frac{\sigma}{1-g}+\nu\right)}. \tag{A.31}$$

Proof: Solve the system of equations (2.42), (2.43), (2.45), (2.46) and (2.55) for $0 \leq t \leq T$ by imposing that $\hat{y}_t = \hat{y}^{ZLB}$, $\pi_t = \pi^{ZLB}$, $\hat{g}_t = \hat{g}^{ZLB}$, $\hat{m}_t = \hat{m}^{ZLB}$, $i_t^M = 0$, $\rho_p = \rho_p^{ZLB}$, and by using the transition probabilities in (2.54) when computing expectations. This gives the following solution for output and inflation

$$\hat{y}^{ZLB} = A_y^{ZLB}\hat{g}^{ZLB} + B_y^{ZLB} \tag{A.32}$$

$$\pi^{ZLB} = \frac{\kappa}{1-\delta\beta_p}\left[\left(\frac{\sigma}{1-g}+\nu\right)\hat{y}^{ZLB} - \frac{\sigma g}{1-g}\hat{g}^{ZLB}\right], \tag{A.33}$$

where the coefficients A_y^{ZLB} and B_y^{ZLB} are defined as

$$A_y^{ZLB}\frac{\Gamma_g\left(1-\delta\right)+\left[\Xi_p\psi_\Pi\left(1-\delta\right)-\delta\Xi_c\right]\frac{\kappa}{\left(1-\beta_p\delta\right)}\frac{\sigma g}{1-g}}{\left(1-\delta\right)\left(1+\Xi_p\psi_Y\right)+\left[\Xi_p\psi_\Pi\left(1-\delta\right)-\delta\Xi_c\right]\frac{\kappa}{1-\beta_p\delta}\left(\frac{\sigma}{1-g}+\nu\right)}$$

$$B_y^{ZLB} \equiv \frac{\Xi_c\left(\frac{1-\beta_c^{ZLB}}{\beta_c^{ZLB}}\right)}{\left(1-\delta\right)\left(1+\Xi_p\psi_Y\right)+\left[\Xi_p\psi_\Pi\left(1-\delta\right)-\delta\Xi_c\right]\frac{\kappa}{1-\beta_p\delta}\left(\frac{\sigma}{1-g}+\nu\right)}.$$

where in the above definitions I have used the fact that $\rho_c^{ZLB} \equiv \frac{1-\beta_c^{ZLB}}{\beta_c^{ZLB}}$. The multiplier on impact is given by

$$\mathcal{M}_0^{ZLB} = \frac{dY_0^{ZLB}}{dG_0^{ZLB}} = \frac{A_y^{ZLB}}{g}.$$

A.5 Three-Agent Model: Proof of Proposition 5

Proposition 5: *If government spending and real reserves are the state variables, then the fiscal and quantitative easing multipliers at the zero lower bound satisfy*

$$\mathcal{M}_0^{G,ZLB} = \frac{1}{g} \frac{\Gamma_g (1-\delta) - \delta \Xi_c \frac{\kappa}{(1-\beta_p \delta)} \frac{\sigma g}{1-g}}{1 - \delta - \delta \Xi_c \frac{\kappa}{1-\beta_p \delta} \left(\frac{\sigma}{1-g} + \nu \right)} \tag{A.34}$$

$$\mathcal{M}_0^{M,ZLB} = \frac{1}{m} \frac{\Xi_p (1-\delta)}{1 - \delta - \delta \Xi_c \frac{\kappa}{1-\beta_p \delta} \left(\frac{\sigma}{1-g} + \nu \right)}. \tag{A.35}$$

Proof: The proof is straightforward and follows the logic of the proof in Proposition 2. Guess that

$$\hat{y}_t = A_y^{G,ZLB} \hat{g}_t + A_y^{M,ZLB} \hat{m}_t + B_y^{ZLB} \tag{A.36}$$
$$\pi_t = A_\pi^{G,ZLB} \hat{g}_t + A_\pi^{M,ZLB} \hat{m}_t + B_\pi^{ZLB}. \tag{A.37}$$

Since $i^M = 0$, and since expectations are computed using δ for next-period values, the multipliers will be similar as in (A.25) and (A.26) with the exceptions that $\phi_\Pi = \phi_Y = 0$, and δ is used instead of ρ_M and ρ_G.

$$A_y^{G,ZLB} = \frac{1}{g} \frac{\Gamma_g (1-\delta) - \delta \Xi_c \frac{\kappa}{(1-\beta_p \delta)} \frac{\sigma g}{1-g}}{1 - \delta - \delta \Xi_c \frac{\kappa}{1-\beta_p \delta} \left(\frac{\sigma}{1-g} + \nu \right)}$$

$$A_\pi^{G,ZLB} = \frac{\kappa}{1-\beta_p \rho_G} \left[\left(\frac{\sigma}{1-g} + \nu \right) A_y^{G,ZLB} - \frac{\sigma g}{1-g} \right]$$

$$A_y^{M,ZLB} = \frac{\Xi_p (1-\delta)}{1 - \delta - \delta \Xi_c \frac{\kappa}{1-\beta_p \delta} \left(\frac{\sigma}{1-g} + \nu \right)}$$

$$A_\pi^{M,ZLB} = \frac{\kappa}{1-\beta_p \delta} \left(\frac{\sigma}{1-g} + \nu \right) A_y^{M,ZLB}$$

and also

$$B_y^{ZLB} = \frac{\Xi_c \left(\frac{1-\beta_c^{ZLB}}{\beta_c^{ZLB}} \right)}{(1-\delta) - \delta \Xi_c \frac{\kappa}{1-\beta_p \delta} \left(\frac{\sigma}{1-g} + \nu \right)}$$

$$B_\pi^{ZLB} = \frac{\kappa}{1-\beta\delta} \left(\frac{\sigma}{1-g} + \nu \right) B_y^{ZLB}$$

The multipliers are then

$$\mathcal{M}_0^G = \frac{A_y^{G,ZLB}}{g}$$
$$\mathcal{M}_0^M = \frac{A_y^{M,ZLB}}{m}.$$

A.6 Three-Agent Model: Proof of Proposition 6

Proposition 6: *Suppose that the government is utilitarian so that social welfare is given by $\mathcal{W} = \eta_p U_p + \eta_c U_c + \eta_h U_h$. Let also β_G be the discount factor of the government. A quadratic approximation of the previous social welfare function around the steady state gives rise to the social welfare function*

$$\mathcal{L} = -\frac{U_C Y}{2}\mathbb{E}_0 \sum_{t=0}^{T} \beta_G^t \left[\sigma\left(1-g\right)\left(\eta_p \hat{c}_{pt}^2 + \eta_c \hat{c}_{ct}^2 + \eta_h \hat{c}_{ht}^2\right) + \nu\left(\eta_p \hat{l}_{pt}^2 + \eta_c \hat{l}_{ct}^2 + \eta_h \hat{l}_{ht}^2\right) + g\zeta \hat{g}_t^2 + \xi_p \pi_t^2\right]. \quad \text{(A.38)}$$

Proof: The model is approximated around a steady state where $C_p = C_c = C_h = C$ and $L_p = L_c = L_h = L$. I first approximate the utility function of the parents without loss of generality.

$$\begin{aligned}
U_p &\simeq U_C C\left[\hat{c}_{pt} + \frac{1-\sigma}{2}\hat{c}_{pt}^2\right] - U_L L\left(\hat{l}_{pt} + \frac{1+\nu}{2}\hat{l}_{pt}^2\right) + U_G G\left(\hat{g}_t + \frac{1-\zeta}{2}\hat{g}_t^2\right) + t.i.p + O\left(\|z\|^3\right) \Rightarrow \\
U_p &\simeq U_C Y\left[\frac{C}{Y}\hat{c}_{pt} + \frac{C}{Y}\frac{1-\sigma}{2}\hat{c}_{pt}^2\right] - U_L L\left(\hat{l}_{pt} + \frac{1+\nu}{2}\hat{l}_{pt}^2\right) + U_G Y\left(\frac{G}{Y}\hat{g}_t + \frac{G}{Y}\frac{1-\zeta}{2}\hat{g}_t^2\right) + t.i.p + O\left(\|z\|^3\right) \Rightarrow \\
U_p &\simeq U_C Y\left[\left(1-g\right)\hat{c}_{pt} + \left(1-g\right)\frac{1-\sigma}{2}\hat{c}_{pt}^2\right] - U_L L\left(\hat{l}_{pt} + \frac{1+\nu}{2}\hat{l}_{pt}^2\right) + U_G Y\left(g\hat{g}_t + g\frac{1-\zeta}{2}\hat{g}_t^2\right) + t.i.p + O\left(\|z\|^3\right) \Rightarrow \\
\eta_p U_p &\simeq U_C Y\left[\eta_p\left(1-g\right)\hat{c}_{pt} + \eta_p\left(1-g\right)\frac{1-\sigma}{2}\hat{c}_{pt}^2\right] - U_L L\left(\eta_p \hat{l}_{pt} + \eta_p\frac{1+\nu}{2}\hat{l}_{pt}^2\right) + U_G Y\left(\eta_p g\hat{g}_t + \eta_p g\frac{1-\zeta}{2}\hat{g}_t^2\right)
\end{aligned} \quad \text{(A.39)}$$

where in the last line of (A.39) I have dropped the terms that do not affect the problems of the agents. In a similar way, the utility functions of the other two types are given by

$$\eta_c U_c \simeq U_C Y\left[\eta_c\left(1-g\right)\hat{c}_{ct} + \eta_c\left(1-g\right)\frac{1-\sigma}{2}\hat{c}_{ct}^2\right] - U_L L\left(\eta_c \hat{l}_{ct} + \eta_c\frac{1+\nu}{2}\hat{l}_{ct}^2\right) + U_G Y\left(\eta_c g\hat{g}_t + \eta_c g\frac{1-\zeta}{2}\hat{g}_t^2\right) \quad \text{(A.40)}$$

$$\eta_h U_h \simeq U_C Y\left[\eta_h\left(1-g\right)\hat{c}_{ht} + \eta_h\left(1-g\right)\frac{1-\sigma}{2}\hat{c}_{ht}^2\right] - U_L L\left(\eta_h \hat{l}_{ht} + \eta_h\frac{1+\nu}{2}\hat{l}_{ht}^2\right) + U_G Y\left(\eta_h g\hat{g}_t + \eta_h g\frac{1-\zeta}{2}\hat{g}_t^2\right) \quad \text{(A.41)}$$

Adding by parts (A.39), (A.40) and (A.41) and using the labor supply optimality condition for each type of household at the steady state $-U_L = U_C w$, where $w = 1$ due to the optimal labor subsidy imposed by the government, we get

$$\mathcal{W} \simeq U_C Y \left[(1-g)\left(\eta_p \hat{c}_{pt} + \eta_c \hat{c}_{ct} + \eta_h \hat{c}_{ht}\right) + \frac{1-\sigma}{2}(1-g)\left(\eta_p \hat{c}_{pt}^2 + \eta_c \hat{c}_{ct}^2 + \eta_h \hat{c}_{ht}^2\right)\right]$$
$$- U_C Y \left[\eta_p \hat{l}_{pt} + \eta_c \hat{l}_{ct} + \eta_h \hat{l}_{ht} + \frac{1+\nu}{2}\left(\eta_p \hat{l}_{pt}^2 + \eta_c \hat{l}_{ct}^2 + \eta_h \hat{l}_{ht}^2\right)\right] + U_C Y \left(g \hat{g}_t + \frac{1-\zeta}{2} g \hat{g}_t^2 \right) \tag{A.42}$$

The goods market clearing condition is approximated as follows

$$\eta_p C_{pt} + \eta_c C_{ct} + \eta_h C_{ht} + G_t = Y_t \left[1 - \frac{\xi_p}{2}\left(\Pi_t - 1\right)^2 \right] \Rightarrow$$

$$\eta_p C \hat{c}_{pt} + \eta_c C \hat{c}_{ct} + \eta_h C \hat{c}_{ht} + G \hat{g}_t + \frac{1}{2}\left(\eta_p C \hat{c}_{pt}^2 + \eta_c C \hat{c}_{ct}^2 + \eta_h C \hat{c}_{ht}^2 + G \hat{g}_t^2\right) = \eta_p L \hat{l}_{pt} + \eta_c L \hat{l}_{ct} + \eta_h L \hat{l}_{ht}$$
$$+ \frac{1}{2}\left(\eta_p L \hat{l}_{pt}^2 + \eta_c L \hat{l}_{ct}^2 + \eta_h L \hat{l}_{ht}^2\right) - Y \frac{\xi_p}{2} \pi_t^2 \stackrel{L=Y}{\Rightarrow}$$

$$\frac{C}{Y}\left(\eta_p \hat{c}_{pt} + \eta_c \hat{c}_{ct} + \eta_h \hat{c}_{ht} + \frac{1}{2}\eta_p \hat{c}_{pt}^2 + \eta_c \frac{1}{2}\hat{c}_{ct}^2 + \eta_h \frac{1}{2}\hat{c}_{ht}^2\right) + \frac{G}{Y}\left(\hat{g}_t + \frac{1}{2}\hat{g}_t^2\right) - \eta_p \hat{l}_{pt} - \eta_c \hat{l}_{ct} - \eta_h \hat{l}_{ht}$$
$$- \frac{1}{2}\eta_p \hat{l}_{pt}^2 - \frac{1}{2}\eta_p \hat{l}_{ct}^2 - \frac{1}{2}\eta_p \hat{l}_{ht}^2 = -\frac{\xi_p}{2}\pi_t \Rightarrow$$

$$(1-g)\left(\eta_p \hat{c}_{pt} + \eta_c \hat{c}_{ct} + \eta_h \hat{c}_{ht} + \frac{1}{2}\eta_p \hat{c}_{pt}^2 + \frac{1}{2}\eta_c \hat{c}_{ct}^2 + \frac{1}{2}\eta_h \hat{c}_{ht}^2\right) + g\left(\hat{g}_t + \frac{1}{2}\hat{g}_t^2\right) - \eta_p \hat{l}_{pt} - \eta_c \hat{l}_{ct} - \eta_h \hat{l}_{ht}$$
$$- \frac{1}{2}\eta_p \hat{l}_{pt}^2 - \frac{1}{2}\eta_p \hat{l}_{ct}^2 - \frac{1}{2}\eta_p \hat{l}_{ht}^2 = -\frac{\xi_p}{2}\pi_t \tag{A.43}$$

If we substitute equation (A.43) into (A.42) we get the social welfare function:

$$\mathcal{W} \simeq -\frac{U_C Y}{2}\left[\sigma(1-g)\left(\eta_p \hat{c}_{pt}^2 + \eta_c \hat{c}_{ct}^2 + \eta_h \hat{c}_{ht}^2\right) + \nu\left(\eta_p \hat{l}_{pt}^2 + \eta_c \hat{l}_{ct}^2 + \eta_h \hat{l}_{ht}^2\right) + g\zeta \hat{g}_t^2 + \xi_p \pi_t^2\right] \tag{A.44}$$

Then, $\mathcal{L}$ is defined as the discounted sum of expected future values of $\mathcal{W}$

$$\mathcal{L} = -\frac{U_C Y}{2}\mathbb{E}_0 \sum_{t=0}^{T} \beta_G^t \left[\sigma(1-g)\left(\eta_p \hat{c}_{pt}^2 + \eta_c \hat{c}_{ct}^2 + \eta_h \hat{c}_{ht}^2\right) + \nu\left(\eta_p \hat{l}_{pt}^2 + \eta_c \hat{l}_{ct}^2 + \eta_h \hat{l}_{ht}^2\right) + g\zeta \hat{g}_t^2 + \xi_p \pi_t^2\right]. \tag{A.45}$$

A.7 Three-Agent Model: Proof of Proposition 7

Proposition 7: *At the zero lower bound, the social welfare function takes the following form*

$$\mathcal{L}^{ZLB} = -\frac{U_C Y}{2}\frac{1-(\beta_G\delta)^{T+1}}{1-\beta_G\delta}\left[\sigma(1-g)\left(\eta_p\hat{c}_p^2+\eta_c\hat{c}_c^2+\eta_h\hat{c}_h^2\right)+\nu\left(\eta_p\hat{l}_p^2+\eta_c\hat{l}_c^2+\eta_h\hat{l}_h^2\right)+g\zeta\hat{g}^2+\xi_p\pi^2\right] \tag{A.46}$$

In addition, there exists a unique solution to the government's problem of choosing optimally $\hat{g}^{*ZLB}$ to maximize the social welfare function subject to the constraints imposed by the behavior of the private sector and the central bank.

Proof: Start with the expression in (A.45). Given that the economy remains at the ZLB for T periods and the probability of remaining at the ZLB for one more period is constant and equal to δ, and the fact that as long as the economy remains at the ZLB all the variables are constant and equal to their ZLB values, the current and future terms inside the sum are just constants that are multiplied by a different power of δ in each period. As a result, the whole sum is a geometric progression that can be simplified as follows:

$$\mathcal{L}^{ZLB} = -\frac{U_C Y}{2}\frac{1-(\beta_G\delta)^{T+1}}{1-\beta_G\delta}\left[\sigma(1-g)\left(\eta_p\hat{c}_p^2+\eta_c\hat{c}_c^2+\eta_h\hat{c}_h^2\right)+\nu\left(\eta_p\hat{l}_p^2+\eta_c\hat{l}_c^2+\eta_h\hat{l}_h^2\right)+g\zeta\hat{g}^2+\xi_p\pi^2\right] \tag{A.47}$$

The government chooses $\hat{g}^{*ZLB}$ to maximize the above social welfare function subject to

$$\pi = \beta_p\delta\pi+\kappa\widehat{\mathrm{mc}} = \beta_p\delta\pi+\kappa\hat{w} = \beta_p\delta\pi+\kappa(\sigma\hat{c}+\nu\hat{y}) \tag{A.48}$$

$$\hat{y} = \delta\hat{y}-\Gamma_g(\delta\hat{g}-\hat{g})-\Xi_c(-\delta\pi-\rho_c)-\Xi_p(\delta\hat{m}-\hat{m}) \tag{A.49}$$

$$\hat{c}_c = \frac{1}{\eta_c}\left(\hat{c}-\eta_p\hat{c}_p-\eta_h\hat{c}_h\right) \tag{A.50}$$

$$\hat{c}_p = \frac{\frac{wL}{C}\frac{\nu+1}{\nu}\left(\frac{\sigma}{1-g}+\nu\right)+\frac{Y}{C}\left[1-w\left(\frac{\sigma}{1-g}+\nu+1\right)\right]}{1+\frac{\sigma}{\nu}\frac{wL}{C}}\hat{y}-\frac{\frac{1}{\nu}\frac{wL}{C}\frac{\sigma g}{1-g}}{1+\frac{\sigma}{\nu}\frac{wL}{C}}\hat{g}+\frac{qB^{CB}}{\eta_p PC}\frac{1}{1+\frac{\sigma}{\nu}\frac{wL}{C}}\hat{m} \tag{A.51}$$

$$\hat{c}_h = \frac{(\nu+1)\dfrac{wL}{C}}{\nu+\sigma\dfrac{wL}{C}}(\sigma\hat{c}+\nu\hat{y}) \tag{A.52}$$

$$\hat{l}_c = \frac{1}{\eta_c}\left(\hat{l} - \eta_p \hat{l}_p - \eta_h \hat{l}_h\right) = \frac{1}{\eta_c}\left(\hat{y} - \eta_p \hat{l}_p - \eta_h \hat{l}_h\right) \tag{A.53}$$

$$\hat{l}_p = \frac{1}{\nu}\left[\sigma \hat{c} + \nu \hat{y} - \sigma \hat{c}_p\right] \tag{A.54}$$

$$\hat{l}_h = \frac{1}{\nu}\left[\sigma \hat{c} + \nu \hat{y} - \sigma \hat{c}_h\right] \tag{A.55}$$

$$\hat{m} = -\psi_\Pi \pi - \psi_Y \hat{y} \tag{A.56}$$

$$\hat{y} = (1-g)\,\hat{c} + g\hat{g}. \tag{A.57}$$

In the above expressions any variable at time t takes its zero lower bound value, but the ZLB superscript is dropped to save on notation. The way to proceed in the above problem is to combine the constraints and make every other variable in the objective function (A.47) a function of $\hat{g}$, so as to have only one optimality condition. Starting with equations (A.48), (A.49), (A.56) and (A.57) we can solve for π and $\hat{y}$ as follows:

$$\hat{y} = A_y^{ZLB}\hat{g} + B_y^{ZLB} \tag{A.58}$$

$$\pi = A_\pi^{ZLB}\hat{g} + B_\pi^{ZLB}, \tag{A.59}$$

where the coefficients are given as

$$A_y^{ZLB} \equiv \frac{\Gamma_g(1-\delta) + \left[\Xi_p \psi_\Pi (1-\delta) - \delta \Xi_c\right] \frac{\kappa}{(1-\beta_p \delta)} \frac{\sigma g}{1-g}}{(1-\delta)\left(1 + \Xi_p \psi_Y\right) + \left[\Xi_p \psi_\Pi (1-\delta) - \delta \Xi_c\right] \frac{\kappa}{1-\beta_p \delta}\left(\frac{\sigma}{1-g} + \nu\right)}$$

$$B_y^{ZLB} \equiv \frac{\Xi_c \left(\frac{1-\beta_c^{ZLB}}{\beta_c^{ZLB}}\right)}{(1-\delta)\left(1 + \Xi_p \psi_Y\right) + \left[\Xi_p \psi_\Pi (1-\delta) - \delta \Xi_c\right] \frac{\kappa}{1-\beta_p \delta}\left(\frac{\sigma}{1-g} + \nu\right)}$$

$$A_\pi^{ZLB} \equiv \frac{\kappa}{1-\beta\delta}\left[\left(\frac{\sigma}{1-g} + \nu\right) A_y^{ZLB} - \frac{\sigma g}{1-g}\right]$$

$$B_\pi^{ZLB} \equiv \frac{\kappa}{1-\beta\delta}\left(\frac{\sigma}{1-g} + \nu\right) B_y^{ZLB}$$

Now that we have expressed output and inflation deviations from steady state as functions of government spending deviations from steady state, we can express all the other variables as functions of $\hat{g}$ because all other variables depend on output and inflation. Starting with the consumption of the hand-to-mouth in (A.52) and using the resource constraint (A.57) and equation (A.58) we have

$$\hat{c}_h = H_1\hat{g} + H_2, \tag{A.60}$$

where

$$H_1 \equiv \frac{(\nu+1)\dfrac{wL}{C}}{\nu+\sigma\dfrac{wL}{C}}\left[\left(\frac{\sigma}{1-g}+\nu\right)A_y^{ZLB}-\frac{\sigma g}{1-g}\right]$$
$$H_2 \equiv \frac{(\nu+1)\dfrac{wL}{C}}{\nu+\sigma\dfrac{wL}{C}}\left(\frac{\sigma}{1-g}+\nu\right)B_y^{ZLB}$$

Next, we can use equations (A.51), (A.56), (A.58) and (A.59) to solve for the consumption deviation from steady state of the parents as a function of government spending deviation:

$$\hat{c}_p = H_3\hat{g} + H_4 \tag{A.61}$$

where

$$H_3 \equiv \left[\frac{\frac{wL}{C}\frac{\nu+1}{\nu}\left(\frac{\sigma}{1-g}+\nu\right)+\frac{Y}{C}\left[1-w\left(\frac{\sigma}{1-g}+\nu+1\right)\right]}{1+\frac{\sigma}{\nu}\frac{wL}{C}}-\frac{qB^{CB}}{\eta_p PC}\frac{1}{1+\frac{\sigma}{\nu}\frac{wL}{C}}\psi_Y\right]A_y^{ZLB}-\frac{\frac{1}{\nu}\frac{wL}{C}\frac{\sigma g}{1-g}}{1+\frac{\sigma}{\nu}\frac{wL}{C}}-\frac{qB^{CB}}{\eta_p PC}\frac{1}{1+\frac{\sigma}{\nu}\frac{wL}{C}}\psi_\Pi A_\pi^{ZLB}$$
$$H_4 \equiv \left[\frac{\frac{wL}{C}\frac{\nu+1}{\nu}\left(\frac{\sigma}{1-g}+\nu\right)+\frac{Y}{C}\left[1-w\left(\frac{\sigma}{1-g}+\nu+1\right)\right]}{1+\frac{\sigma}{\nu}\frac{wL}{C}}-\frac{qB^{CB}}{\eta_p PC}\frac{1}{1+\frac{\sigma}{\nu}\frac{wL}{C}}\psi_Y\right]B_y^{ZLB}-\frac{qB^{CB}}{\eta_p PC}\frac{1}{1+\frac{\sigma}{\nu}\frac{wL}{C}}\psi_\Pi B_\pi^{ZLB}$$

Regarding the consumption function of the children we need to combine equations (A.50), (A.57), (A.58), (A.60) and (A.61). Then, we get:

$$\hat{c}_c = H_5\hat{g} + H_6 \tag{A.62}$$

where

$$H_5 \equiv \frac{1}{\eta_c}\left(\frac{A_y^{ZLB}-g}{1-g}-\eta_p H_3-\eta_h H_1\right)$$
$$H_6 \equiv \frac{1}{\eta_c}\left(\frac{B_y^{ZLB}}{1-g}-\eta_p H_4-\eta_h H_2\right).$$

Next, we combine equations (A.55), (A.57), (A.58) and (A.60) and get:

$$\hat{l}_h = H_7 \hat{g} + H_8, \tag{A.63}$$

where

$$\begin{aligned} H_7 &\equiv \frac{1}{\nu} \left[\left(\frac{\sigma}{1-g} + \nu \right) A_y^{ZLB} - \frac{\sigma g}{1-g} - \sigma H_1 \right] \\ H_8 &\equiv \frac{1}{\nu} \left[\left(\frac{\sigma}{1-g} + \nu \right) B_y^{ZLB} - \sigma H_2 \right]. \end{aligned}$$

In the same spirit, equations (A.54), (A.57), (A.58) and (A.61) imply that

$$\hat{l}_p = H_9 \hat{g} + H_{10}, \tag{A.64}$$

where

$$\begin{aligned} H_9 &\equiv \frac{1}{\nu} \left[\left(\frac{\sigma}{1-g} + \nu \right) A_y^{ZLB} - \frac{\sigma g}{1-g} - \sigma H_3 \right] \\ H_{10} &\equiv \frac{1}{\nu} \left[\left(\frac{\sigma}{1-g} + \nu \right) B_y^{ZLB} - \sigma H_4 \right]. \end{aligned}$$

Finally, using equations (A.53), (A.58), (A.63) and (A.64), we can rewrite the labor supply of the children as

$$\hat{l}_c = H_{11} \hat{g} + H_{12}, \tag{A.65}$$

where

$$\begin{aligned} H_{11} &\equiv \frac{1}{\eta_c} \left(A_y^{ZLB} - \eta_p H_9 - \eta_h H_7 \right) \\ H_{12} &\equiv \frac{1}{\eta_c} \left(B_y^{ZLB} - \eta_p H_{10} - \eta_h H_8 \right). \end{aligned}$$

Now that we have expressed all variables in (A.47) as functions of $\hat{g}$ only, we can take the first order condition with respect to $\hat{g}$, using the chain rule of differentiation, and solve for $\hat{g}^{*ZLB}$:

$$\sigma (1-g) \left(\eta_p \hat{c}_p \frac{\partial \hat{c}_p}{\partial \hat{g}} + \eta_c \hat{c}_c \frac{\partial \hat{c}_c}{\partial \hat{g}} + \eta_h \hat{c}_h \frac{\partial \hat{c}_h}{\partial \hat{g}} \right) + \nu \left(\eta_p \hat{l}_p \frac{\partial \hat{l}_p}{\partial \hat{g}} + \eta_c \hat{l}_c \frac{\partial \hat{l}_c}{\partial \hat{g}} + \eta_h \hat{l}_h \frac{\partial \hat{l}_h}{\partial \hat{g}} \right) + g \zeta \hat{g} + \xi_p \pi \frac{\partial \pi}{\partial \hat{g}} = 0, \tag{A.66}$$

where all the partial derivatives are taken using equations (A.59) and (A.60)-(A.65). Then, we can show that

$$\hat{g}^{*ZLB} = \frac{H_{13}}{H_{14}}, \tag{A.67}$$

where

$$H_{13} \equiv -\sigma (1-g) \left(\eta_c H_5 H_6 + \eta_p H_3 H_4 + \eta_h H_1 H_2\right) - \nu \left(\eta_c H_{11} H_{12} + \eta_p H_9 H_{10} + \eta_h H_7 H_8\right) - \xi_p A_{\pi}^{ZLB} B_{\pi}^{ZLB}$$
$$H_{14} \equiv \sigma (1-g) \left(\eta_c H_5^2 + \eta_p H_3^2 + \eta_h H_1^2\right) + \nu \left(\eta_c H_{11}^2 + \eta_p H_9^2 + \eta_h H_7^2\right) + g\zeta + \xi_p \left(A_{\pi}^{ZLB}\right)^2.$$

A.8 Three-Agent Model: Proof of Proposition 8

Proposition 8: *There exists a unique solution to the government's problem of choosing optimally $\hat{g}^{*ZLB}$ and $\hat{m}^{*ZLB}$ to maximize the social welfare function in (2.60) subject to the constraints imposed by the behavior of the private sector.*

Proof: Suppose now that the government chooses optimally both $\hat{g}^{*ZLB}$ and $\hat{m}^{*ZLB}$ to maximize

$$\mathcal{L}^{ZLB} = -\frac{U_c Y}{2}\frac{1-(\beta_G\delta)^{T+1}}{1-\beta_G\delta}\left[\sigma(1-g)\left(\eta_p\hat{c}_p^2+\eta_c\hat{c}_c^2+\eta_h\hat{c}_h^2\right)+\nu\left(\eta_p\hat{l}_p^2+\eta_c\hat{l}_c^2+\eta_h\hat{l}_h^2\right)+g\zeta\hat{g}^2+\xi_p\pi^2\right] \tag{A.68}$$

subject to

$$\pi = \beta_p\delta\pi+\kappa\widehat{\mathrm{mc}} = \beta_p\delta\pi+\kappa\hat{w} = \beta_p\delta\pi+\kappa\left(\sigma\hat{c}+\nu\hat{y}\right) \tag{A.69}$$

$$\hat{y} = \delta\hat{y}-\Gamma_g\left(\delta\hat{g}-\hat{g}\right)-\Xi_c\left(-\delta\pi-\rho_c\right)-\Xi_p\left(\delta\hat{m}-\hat{m}\right) \tag{A.70}$$

$$\hat{c}_c = \frac{1}{\eta_c}\left(\hat{c}-\eta_p\hat{c}_p-\eta_h\hat{c}_h\right) \tag{A.71}$$

$$\hat{c}_p = \frac{\frac{wL}{C}\frac{\nu+1}{\nu}\left(\frac{\sigma}{1-g}+\nu\right)+\frac{Y}{C}\left[1-w\left(\frac{\sigma}{1-g}+\nu+1\right)\right]}{1+\frac{\sigma}{\nu}\frac{wL}{C}}\hat{y}-\frac{\frac{1}{\nu}\frac{wL}{C}\frac{\sigma g}{1-g}}{1+\frac{\sigma}{\nu}\frac{wL}{C}}\hat{g}+\frac{qB^{CB}}{\eta_p PC}\frac{1}{1+\frac{\sigma}{\nu}\frac{wL}{C}}\hat{m} \tag{A.72}$$

$$\hat{c}_h = \frac{(\nu+1)\dfrac{wL}{C}}{\nu+\sigma\dfrac{wL}{C}}\left(\sigma\hat{c}+\nu\hat{y}\right) \tag{A.73}$$

$$\hat{l}_c = \frac{1}{\eta_c}\left(\hat{l}-\eta_p\hat{l}_p-\eta_h\hat{l}_h\right) = \frac{1}{\eta_c}\left(\hat{y}-\eta_p\hat{l}_p-\eta_h\hat{l}_h\right) \tag{A.74}$$

$$\hat{l}_p = \frac{1}{\nu}\left[\sigma\hat{c}+\nu\hat{y}-\sigma\hat{c}_p\right] \tag{A.75}$$

$$\hat{l}_h = \frac{1}{\nu}\left[\sigma\hat{c}+\nu\hat{y}-\sigma\hat{c}_h\right] \tag{A.76}$$

$$\hat{y} = (1-g)\,\hat{c}+g\hat{g}. \tag{A.77}$$

In this part I proceed in the same way as in the previous section using the constraints to make all variables inside the expression given by (A.68) functions of the choice variables $\hat{g}$ and $\hat{m}$. Specifically, equations (A.69), (A.70) and (A.77) imply that

$$\hat{y} = A_y^g\hat{g}+A_y^m\hat{m}+B_y \tag{A.78}$$

$$\pi = A_\pi^g\hat{g}+A_\pi^m\hat{m}+B_\pi, \tag{A.79}$$

where the coefficients are given as

$$
\begin{aligned}
A_y^g &\equiv \frac{\Gamma_g\left(1-\delta\right)-\delta\Xi_c\dfrac{\kappa}{\left(1-\beta_p\delta\right)}\dfrac{\sigma g}{1-g}}{\left(1-\delta\right)-\delta\Xi_c\dfrac{\kappa}{1-\beta_p\delta}\left(\dfrac{\sigma}{1-g}+\nu\right)}\\
A_y^m &\equiv \frac{\Xi_p\left(1-\delta\right)}{\left(1-\delta\right)-\delta\Xi_c\dfrac{\kappa}{1-\beta_p\delta}\left(\dfrac{\sigma}{1-g}+\nu\right)}\\
B_y &\equiv \frac{\Xi_c\left(\frac{1-\beta_c^{ZLB}}{\beta_c^{ZLB}}\right)}{\left(1-\delta\right)-\delta\Xi_c\dfrac{\kappa}{1-\beta_p\delta}\left(\dfrac{\sigma}{1-g}+\nu\right)}\\
A_\pi^g &\equiv \frac{\kappa}{1-\beta\delta}\left[\left(\frac{\sigma}{1-g}+\nu\right)A_y^g-\frac{\sigma g}{1-g}\right]\\
A_\pi^m &\equiv \frac{\kappa}{1-\beta\delta}\left(\frac{\sigma}{1-g}+\nu\right)A_y^m\\
B_\pi &\equiv \frac{\kappa}{1-\beta\delta}\left(\frac{\sigma}{1-g}+\nu\right)B_y
\end{aligned}
$$

If we combine equations (A.73), (A.77) and (A.78), the consumption of the constrained households is

$$
\hat{c}_h = J_1\hat{g} + J_2\hat{m} + J_3, \tag{A.80}
$$

where

$$
\begin{aligned}
J_1 &\equiv \frac{\left(\nu+1\right)\dfrac{wL}{C}}{\nu+\sigma\dfrac{wL}{C}}\left[\left(\frac{\sigma}{1-g}+\nu\right)A_y^g-\frac{\sigma g}{1-g}\right]\\
J_2 &\equiv \frac{\left(\nu+1\right)\dfrac{wL}{C}}{\nu+\sigma\dfrac{wL}{C}}\left(\frac{\sigma}{1-g}+\nu\right)A_y^m\\
J_3 &\equiv \frac{\left(\nu+1\right)\dfrac{wL}{C}}{\nu+\sigma\dfrac{wL}{C}}\left(\frac{\sigma}{1-g}+\nu\right)B_y.
\end{aligned}
$$

The consumption function of the parents is now given by the combination of equations (A.72) and (A.78)

$$
\hat{c}_p = J_4\hat{g} + J_5\hat{m} + J_6 \tag{A.81}
$$

where

$$
\begin{aligned}
J_4 &\equiv \frac{\frac{wL}{C}\frac{\nu+1}{\nu}\left(\frac{\sigma}{1-g}+\nu\right)+\frac{Y}{C}\left[1-w\left(\frac{\sigma}{1-g}+\nu+1\right)\right]}{1+\frac{\sigma}{\nu}\frac{wL}{C}}A_y^g-\frac{\frac{1}{\nu}\frac{wL}{C}\frac{\sigma g}{1-g}}{1+\frac{\sigma}{\nu}\frac{wL}{C}} \\
J_5 &\equiv \frac{\frac{wL}{C}\frac{\nu+1}{\nu}\left(\frac{\sigma}{1-g}+\nu\right)+\frac{Y}{C}\left[1-w\left(\frac{\sigma}{1-g}+\nu+1\right)\right]}{1+\frac{\sigma}{\nu}\frac{wL}{C}}A_y^m+\frac{qB^{CB}}{\eta_p PC}\frac{1}{1+\frac{\sigma}{\nu}\frac{wL}{C}} \\
J_6 &\equiv \frac{\frac{wL}{C}\frac{\nu+1}{\nu}\left(\frac{\sigma}{1-g}+\nu\right)+\frac{Y}{C}\left[1-w\left(\frac{\sigma}{1-g}+\nu+1\right)\right]}{1+\frac{\sigma}{\nu}\frac{wL}{C}}B_y
\end{aligned}
$$

Then we can use equations (A.71), (A.77), (A.78), (A.80), and (A.81) to solve for the consumption function of the children as follows

$$
\hat{c}_c = J_7\hat{g} + J_8\hat{m} + J_9, \tag{A.82}
$$

where

$$
\begin{aligned}
J_7 &\equiv \frac{1}{\eta_c}\left(\frac{A_y^g-g}{1-g}-\eta_p J_4-\eta_h J_1\right) \\
J_8 &\equiv \frac{1}{\eta_c}\left(\frac{A_y^m}{1-g}-\eta_p J_5-\eta_h J_2\right) \\
J_9 &\equiv \frac{1}{\eta_c}\left(\frac{B_y}{1-g}-\eta_p J_6-\eta_h J_3\right).
\end{aligned}
$$

Next, we can solve for the labor supply function of the hand-to-mouth consumers by combining equations (A.76), (A.77), (A.78) and (A.80)

$$
\hat{l}_h = J_{10}\hat{g} + J_{11}\hat{m} + J_{12}, \tag{A.83}
$$

where

$$
\begin{aligned}
J_{10} &\equiv \frac{1}{\nu}\left[\left(\frac{\sigma}{1-g}+\nu\right)A_y^g-\frac{\sigma g}{1-g}-\sigma J_1\right] \\
J_{11} &\equiv \frac{1}{\nu}\left[\left(\frac{\sigma}{1-g}+\nu\right)A_y^m-\sigma J_2\right] \\
J_{12} &\equiv \frac{1}{\nu}\left[\left(\frac{\sigma}{1-g}+\nu\right)B_y-\sigma J_3\right].
\end{aligned}
$$

In the same spirit, we can combine equations (A.72), (A.75), (A.77), and (A.81) and express the labor supply function of the parents as follows

$$\hat{l}_p = J_{13}\hat{g} + J_{14}\hat{m} + J_{15}, \tag{A.84}$$

where

$$\begin{aligned}
J_{13} &\equiv \frac{1}{\nu}\left[\left(\frac{\sigma}{1-g}+\nu\right)A_y^g - \frac{\sigma g}{1-g} - \sigma J_4\right] \\
J_{14} &\equiv \frac{1}{\nu}\left[\left(\frac{\sigma}{1-g}+\nu\right)A_y^m - \sigma J_5\right] \\
J_{15} &\equiv \frac{1}{\nu}\left[\left(\frac{\sigma}{1-g}+\nu\right)B_y - \sigma J_6\right].
\end{aligned}$$

Finally, using equations (A.74), (A.78), (A.83) and (A.84) we can solve for the labor supply function of the children as follows

$$\hat{l}_c = J_{16}\hat{g} + J_{17}\hat{m} + J_{18}, \tag{A.85}$$

where

$$\begin{aligned}
J_{16} &\equiv \frac{1}{\eta_c}\left(A_y^g - \eta_p J_{13} - \eta_h J_{10}\right) \\
J_{17} &\equiv \frac{1}{\eta_c}\left(A_y^m - \eta_p J_{14} - \eta_h J_{11}\right) \\
J_{18} &\equiv \frac{1}{\eta_c}\left(B_y - \eta_p J_{15} - \eta_h J_{12}\right).
\end{aligned}$$

Now that we have expressed all variables inside (A.68) as functions of $\hat{g}$ and $\hat{m}$ we can again maximize the social welfare function with respect to those two choice variables. This will result in a linear system of two optimality conditions in the two unknown optimal choices, which can be solved analytically. The first order optimality condition with respect to $\hat{g}$ is

$$\sigma(1-g)\left(\eta_p\hat{c}_p\frac{\partial\hat{c}_p}{\partial\hat{g}} + \eta_c\hat{c}_c\frac{\partial\hat{c}_c}{\partial\hat{g}} + \eta_h\hat{c}_h\frac{\partial\hat{c}_h}{\partial\hat{g}}\right) + \nu\left(\eta_p\hat{l}_p\frac{\partial\hat{l}_p}{\partial\hat{g}} + \eta_c\hat{l}_c\frac{\partial\hat{l}_c}{\partial\hat{g}} + \eta_h\hat{l}_h\frac{\partial\hat{l}_h}{\partial\hat{g}}\right) + g\zeta\hat{g} + \xi_p\pi\frac{\partial\pi}{\partial\hat{g}} = 0, \tag{A.86}$$

After some algebraic manipulations, the previous condition can be written as

$$\hat{g}^{*ZLB} = J_{19} + J_{20}\hat{m}^{*ZLB}, \tag{A.87}$$

where

$$J_{19} \equiv -\frac{\sigma\left(1-g\right)\left(\eta_c J_7 J_9 + \eta_p J_4 J_6 + \eta_h J_1 J_3\right) + \nu\left(\eta_c J_{16} J_{18} + \eta_p J_{13} J_{15} + \eta_h J_{10} J_{12}\right) + \xi_p A_\pi^g B_\pi}{\sigma\left(1-g\right)\left(\eta_c J_7^2 + \eta_p J_4^2 + \eta_h J_1^2\right) + \nu\left(\eta_c J_{16}^2 + \eta_p J_{13}^2 + \eta_h J_{10}^2\right) + g\zeta + \xi_p\left(A_\pi^g\right)^2}$$

$$J_{20} \equiv -\frac{\sigma\left(1-g\right)\left(\eta_c J_7 J_8 + \eta_p J_4 J_5 + \eta_h J_1 J_2\right) + \nu\left(\eta_c J_{16} J_{17} + \eta_p J_{13} J_{14} + \eta_h J_{10} J_{11}\right) + \xi_p A_\pi^g A_\pi^m}{\sigma\left(1-g\right)\left(\eta_c J_7^2 + \eta_p J_4^2 + \eta_h J_1^2\right) + \nu\left(\eta_c J_{16}^2 + \eta_p J_{13}^2 + \eta_h J_{10}^2\right) + g\zeta + \xi_p\left(A_\pi^g\right)^2}.$$

In the same spirit, the first order optimality condition with respect to $\hat{m}$ is

$$\sigma\left(1-g\right)\left(\eta_p \hat{c}_p \frac{\partial \hat{c}_p}{\partial \hat{m}} + \eta_c \hat{c}_c \frac{\partial \hat{c}_c}{\partial \hat{m}} + \eta_h \hat{c}_h \frac{\partial \hat{c}_h}{\partial \hat{m}}\right) + \nu\left(\eta_p \hat{l}_p \frac{\partial \hat{l}_p}{\partial \hat{m}} + \eta_c \hat{l}_c \frac{\partial \hat{l}_c}{\partial \hat{m}} + \eta_h \hat{l}_h \frac{\partial \hat{l}_h}{\partial \hat{m}}\right) + \xi_p \pi \frac{\partial \pi}{\partial \hat{m}} = 0, \tag{A.88}$$

After some algebraic manipulations, the previous condition can be written as

$$\hat{m}^{*ZLB} = J_{21} + J_{22}\hat{g}^{*ZLB}, \tag{A.89}$$

where

$$J_{21} \equiv -\frac{\sigma\left(1-g\right)\left(\eta_c J_8 J_9 + \eta_p J_5 J_6 + \eta_h J_2 J_3\right) + \nu\left(\eta_c J_{17} J_{18} + \eta_p J_{14} J_{15} + \eta_h J_{11} J_{12}\right) + \xi_p A_\pi^m B_\pi}{\sigma\left(1-g\right)\left(\eta_c J_8^2 + \eta_p J_5^2 + \eta_h J_2^2\right) + \nu\left(\eta_c J_{17}^2 + \eta_p J_{14}^2 + \eta_h J_{11}^2\right) + \xi_p\left(A_\pi^m\right)^2}$$

$$J_{22} \equiv -\frac{\sigma\left(1-g\right)\left(\eta_c J_7 J_8 + \eta_p J_4 J_5 + \eta_h J_1 J_2\right) + \nu\left(\eta_c J_{16} J_{17} + \eta_p J_{13} J_{14} + \eta_h J_{10} J_{11}\right) + \xi_p A_\pi^m A_\pi^g}{\sigma\left(1-g\right)\left(\eta_c J_8^2 + \eta_p J_5^2 + \eta_h J_2^2\right) + \nu\left(\eta_c J_{17}^2 + \eta_p J_{14}^2 + \eta_h J_{11}^2\right) + \xi_p\left(A_\pi^m\right)^2}.$$

If we combine equations (A.87) and (A.89) we can solve for the optimal $\hat{m}^{*ZLB}$ and $\hat{g}^{*ZLB}$ as follows

$$\hat{m}^{*ZLB} = \frac{J_{21} + J_{19}J_{22}}{1 - J_{20}J_{22}} \tag{A.90}$$

$$\hat{g}^{*ZLB} = J_{19} + J_{20}\frac{J_{21} + J_{19}J_{22}}{1 - J_{20}J_{22}}. \tag{A.91}$$

A.9 HANK Model: Proof of Proposition 9

Proposition 9: *The commercial bank's value function is linear in net worth and satisfies*

$$V_{bt}\left(N_{bt}\right) = \Sigma_t N_{bt} \;\text{ with }\; \Sigma_t = 1. \tag{A.92}$$

Proof: The commercial bank's problem can be expressed as follows

$$V_{bt}\left(N_{bt}\right) = \max_{M_{bt}, D_{bt}} \left\{ \frac{1}{1+r_{t+1}} \left[\left(1-\theta_b\right) N_{bt+1} + \theta_b V_{bt+1}\left(N_{bt+1}\right)\right] \right\} \tag{A.93}$$

subject to

$$D_{bt} = M_{bt} - N_{bt} \tag{A.94}$$

$$N_{bt} = \left(1+r_t\right) M_{bt-1} - \left(1 + r_t^D + \xi_D\right) D_{bt-1}. \tag{A.95}$$

Guess that the value function of the bank is linear in net worth

$$V_{bt}\left(N_{bt}\right) = \Sigma_t N_{bt}. \tag{A.96}$$

Using the previous guess and the constraints (A.94) and (A.95) we can rewrite equation (A.93) as follows

$$V_{bt}\left(N_{bt}\right) = \max_{M_{bt}, D_{bt}} \left\{ \frac{1}{1+r_{t+1}} \left(1-\theta_b+\theta_b\Sigma_{t+1}\right) \left[\left(r_{t+1} - r_{t+1}^D - \xi_D\right) M_{bt} + \left(1 + r_t^D + \xi_D\right) N_{bt}\right] \right\} \tag{A.97}$$

The optimality condition with respect to M_{bt} is

$$\frac{1}{1+r_{t+1}} \left(1-\theta_b+\theta_b\Sigma_{t+1}\right) \left(r_{t+1} - r_{t+1}^D - \xi_D\right) = 0 \Rightarrow r_{t+1} = r_{t+1}^D + \xi_D. \tag{A.98}$$

Finally we need to determine the coefficient Σ_t in the bank's value function. I start from equation (A.93) and use the guess (A.96) on both sides:

$$\begin{aligned}
\Sigma_t N_{bt} &= \frac{1}{1+r_{t+1}} \left(1-\theta_b+\theta_b\Sigma_{t+1}\right) \left[\left(r_{t+1} - r_{t+1}^D - \xi_D\right) M_{bt} + \left(1 + r_{t+1}^D + \xi_D\right) N_{bt}\right] \overset{(A.95)}{\Longrightarrow} \\
\Sigma_t N_{bt} &= \frac{1}{1+r_{t+1}} \left(1-\theta_b+\theta_b\Sigma_{t+1}\right) \left(1 + r_{t+1}^D + \xi_D\right) N_{bt} \overset{(A.98)}{\Longrightarrow} \\
\Sigma_t &= 1-\theta_b+\theta_b\Sigma_{t+1} \Longrightarrow \Sigma_t = 1.
\end{aligned} \tag{A.99}$$

A.10 HANK Model: Proof of Proposition 10

Proposition 10: *The investment bank's value function is linear in net worth and satisfies*

$$V_{nt}^{MF}\left(N_{nt}^{MF}\right) = \Sigma_t^{MF} N_{nt}^{MF} \tag{A.100}$$

$$\Sigma_t^{MF} = \frac{\lambda_K}{\lambda_K - \frac{1}{1+r_{t+1}}\left(1-\theta^{MF}+\theta^{MF}\Sigma_{t+1}^{MF}\right)\left(r_{t+1}^K - r_{t+1}^F\right)} \frac{1+r_{t+1}^F-\xi_N}{1+r_{t+1}}\left(1-\theta^{MF}+\theta^{MF}\Sigma_{t+1}^{MF}\right). \tag{A.101}$$

Proof: The problem of the investment bank can be expressed as follows

$$V_{nt}^{MF}\left(N_{nt}^{MF}\right) = \max_{K_{nt}^{MF}, B_{nt}^{MF}, F_{nt}^{MF}} \left\{\frac{1}{1+r_{t+1}}\left[\left(1-\theta^{MF}\right)N_{nt+1}^{MF} + \theta^{MF} V_{nt+1}^{MF}\left(N_{nt+1}^{MF}\right)\right]\right\} \tag{A.102}$$

subject to

$$Q_t K_{nt}^{MF} + q_t B_{nt}^{MF} = F_{nt}^{MF} + N_{nt}^{MF}. \tag{A.103}$$

$$N_{nt}^{MF} = \left[\left(1+r_t^K\right) - \left(1+r_t^F\right)\right] Q_{t-1} K_{nt-1}^{MF} + \left[\left(1+r_t^B\right) - \left(1+r_t^F\right)\right] q_{t-1} B_{nt-1}^{MF} + \left(1+r_t^F-\xi_N\right) N_{nt-1}^{MF} \tag{A.104}$$

$$V_{nt}^{MF}\left(N_{nt}^{MF}\right) \geq \lambda_K Q_t K_{nt}^{MF} + \lambda_B q_t B_{nt}^{MF}. \tag{A.105}$$

I guess that the value function of the bank is linear in net worth

$$V_{nt}^{MF}\left(N_{nt}^{MF}\right) = \Sigma_t^{MF} N_{nt}^{MF}. \tag{A.106}$$

Using the previous guess we can rewrite equation (A.102) as follows

$$\Sigma_t^{MF} N_t^{MF} = \max\left\{\frac{1}{1+r_{t+1}}\left(1-\theta^{MF}+\theta^{MF}\Sigma_{t+1}^{MF}\right) N_{t+1}^{MF}\right\} \tag{A.107}$$

Let $\mathcal{L}^{MF}$ be the Lagrange function for the previous constrained optimization problem. Let also Θ_t be the Lagrange multiplier. Then we can express the optimization problem as

$$\mathcal{L}^{MF} = \Sigma_t^{MF} N_t^{MF} + \Theta_t \left[V_{nt}^{MF}\left(N_{nt}^{MF}\right) - \lambda_K Q_t K_{nt}^{MF} - \lambda_B q_t B_{nt}^{MF} \right] \stackrel{(A.106)}{\Longrightarrow}$$
$$\mathcal{L}^{MF} = (1+\Theta_t)\, \Sigma_t^{MF} N_t^{MF} - \Theta_t \left[\lambda_K Q_t K_{nt}^{MF} + \lambda_B q_t B_{nt}^{MF} \right] \stackrel{(A.107)}{\Longrightarrow}$$
$$\mathcal{L}^{MF} = (1+\Theta_t)\, \frac{1}{1+r_{t+1}} \left(1-\theta^{MF} + \theta^{MF}\Sigma_{t+1}^{MF}\right) N_{nt+1}^{MF} - \Theta_t \left[\lambda_K Q_t K_{nt}^{MF} + \lambda_B q_t B_{nt}^{MF} \right], \tag{A.108}$$

where N_{nt+1}^{MF} is just equation (A.104) written one period ahead and F_{nt}^{MF} is given by equation (A.103). The optimality conditions with respect to $\left\{K_{nt}^{MF}, B_{nt}^{MF}\right\}$ are

$$(1+\Theta_t)\, \frac{1}{1+r_{t+1}} \left(1-\theta^{MF} + \theta^{MF}\Sigma_{t+1}^{MF}\right) \left(r_{t+1}^{K} - r_{t+1}^{F}\right) = \Theta_t \lambda_K \tag{A.109}$$
$$(1+\Theta_t)\, \frac{1}{1+r_{t+1}} \left(1-\theta^{MF} + \theta^{MF}\Sigma_{t+1}^{MF}\right) \left(r_{t+1}^{B} - r_{t+1}^{F}\right) = \Theta_t \lambda_B \tag{A.110}$$

By dividing by parts equations (A.109) and (A.110) we get

$$1 + r_{t+1}^{B} = \frac{\lambda_B}{\lambda_K}\left(1 + r_{t+1}^{K}\right) + \left(1 - \frac{\lambda_B}{\lambda_K}\right)\left(1 + r_{t+1}^{F}\right) \tag{A.111}$$

Finally we need to determine Σ_t. I start from equation (A.107). Let $\Delta_{t+1} \equiv \frac{1}{1+r_{t+1}}\left(1-\theta^{MF} + \theta^{MF}\Sigma_{t+1}^{MF}\right)$. Then

$$\Sigma_t^{MF} N_{nt}^{MF} = \Delta_{t+1} N_{t+1}^{MF} \stackrel{(A.104)}{\Longrightarrow}$$

$$\Sigma_t^{MF} N_{nt}^{MF} = \Delta_{t+1} \left\{ \left[\left(1 + r_{t+1}^{K}\right) - \left(1 + r_{t+1}^{F}\right) \right] Q_t K_{nt}^{MF} + \left[\left(1 + r_{t+1}^{B}\right) - \left(1 + r_{t+1}^{F}\right) \right] q_t B_{nt}^{MF} + \left(1 + r_{t+1}^{F} - \xi_N\right) N_{nt}^{MF} \right\} \stackrel{(A.109-110)}{\Longrightarrow}$$

$$\Sigma_t^{MF} N_{nt}^{MF} = \Delta_{t+1} \left[\frac{1}{\Delta_{t+1}} \frac{\Theta_t}{1+\Theta_t} \lambda_K Q_t K_{nt}^{MF} + \frac{1}{\Delta_{t+1}} \frac{\Theta_t}{1+\Theta_t} \lambda_B q_t B_{nt}^{MF} + \left(1 + r_{t+1}^{F} - \xi_N\right) N_{nt}^{MF} \right] \stackrel{(A.105),(A.106)}{\Longrightarrow}$$

$$\Sigma_t^{MF} N_{nt}^{MF} = \frac{\Theta_t}{1+\Theta_t} \Sigma_t N_{nt}^{MF} + \frac{1}{1+r_{t+1}} \left(1-\theta^{MF} + \theta^{MF}\Sigma_{t+1}^{MF}\right) \left(1 + r_{t+1}^{F} - \xi_N\right) N_{nt}^{MF} \Longrightarrow$$

$$\Sigma_t^{MF} N_{nt}^{MF} = (1+\Theta_t)\, \frac{1}{1+r_{t+1}} \left(1-\theta^{MF} + \theta^{MF}\Sigma_{t+1}^{MF}\right) \left(1 + r_{t+1}^{F} - \xi_N\right) N_{nt}^{MF} \stackrel{(A.109)}{\Longrightarrow}$$

$$\Sigma_t^{MF} = \frac{\lambda_K}{\lambda_K - \frac{1}{1+r_{t+1}}\left(1-\theta^{MF} + \theta^{MF}\Sigma_{t+1}^{MF}\right)\left(r_{t+1}^{K} - r_{t+1}^{F}\right)} \frac{1 + r_{t+1}^{F} - \xi_N}{1+r_{t+1}} \left(1-\theta^{MF} + \theta^{MF}\Sigma_{t+1}^{MF}\right).$$

B HANK Model: Calibration and Computational Method

B.1 Calibration of the HANK Model

The HANK model is calibrated over the period 2008-2021 using quarterly data for the US economy.

Households: The households are split into two categories: those with a high discount factor and those with a low discount factor. Discount factor heterogeneity is used to hit the following calibration targets: i) the high discount factor is set equal to $\beta^H = 0.9903$ so that $D = 2.401Y$, which is the average value of deposits to GDP according to the Flow of Funds data, and ii) the low discount factor is set equal to $\beta^L = 0.9592$ so as to match the size of the hand-to-mouth households, which is equal to 8% in the SCF of 2019. The parameter measuring the relative weight of labor disutility is set equal to $\mu_L = 1.579$ so as to satisfy the steady state version of the WNKPC given by (3.9). The pivot adjustment cost parameter is free and is set equal to $\chi_0 = 0.01$. The scale adjustment cost parameter is set equal to $\chi_1 = 45.336$ so as to match the average value of total financial assets to GDP for the period 2008-2021 according to the Flow of Funds data without deposits, which is equal to $A = 13.349Y$. The curvature adjustment cost parameter is set equal to $\chi_2 = 1.998$ so as to match the fraction of wealthy hand-to-mouth households which is equal to 14% in the SCF of 2019.[9] The labor productivity process e_t is assumed to follow an $AR(1)$ process with persistence $\rho_e = 0.966$ and standard deviation of the shocks equal to $\sigma_e = 0.92$ as in McKay et al. (2016). The labor productivity process is discretized as a Markov chain with nine nodes. Finally, the upper bound on the grid for the illiquid savings choice of the households is set equal to $\bar{a} = 38.741$ so as to match the average wealth share of the top 10% as given by the WID for the period 2008-2021.[10]

Labor Union: The elasticity of substitution between different types of labor is set equal to $\varepsilon_w = \varepsilon_p = 9$ and the wage adjustment cost parameter is set equal to $\xi_w = \xi_p = 1486.77$ as in Kyriazis (2022). The steady state amount of labor is normalized to $L = 1$.

Investment Bank: The divertible fraction of foreign capital claims is set equal to $\lambda_K = 0.345$, as in Gertler and Karadi (2011). Then, the divertible fraction of foreign government bonds is set equal to $\lambda_B = 0.0163$ so as to satisfy the steady state version of (3.29), given the set value on the bank rate $r^F = 0.175\%$. The steady sate real interest rate on government bonds used is equal to $r^B = 0.867\%$per annum[11] using the average real return for the period 2008-2021 on 10-year TIPS as provided by the FRED website. The parameter controlling the magnitude of the transfer received by new banks entering is set to $\omega = 0.001$. The bank survival rate and

[9]The SCF can be found here https://www.federalreserve.gov/econres/scfindex.htm

[10]See https://wid.world/data/

[11]The size of r^F cannot exceed the quarterly value of r^B, otherwise $\lambda_B < 0$.

the management cost are jointly determined to satisfy the steady state versions of equations (3.31) and (3.33). They take the values $\theta^{MF} = 0.9966$, $\omega = 0.0011$, and $\xi_N = 0.0273$.

Commercial Bank: The unit cost of intermediation is set equal to $\xi_D = 0$.

Investment Fund: The unit cost of intermediation is set equal to $\xi_A = rF$ at the steady state so that the illiquid return in (3.21) is determined only by the after tax aggregate profits over savings.

Intermediate Goods Producers: Steady state output is normalized to $Y = 1$, and quarterly steady state capital is set equal to $K = 14.08Y$ as implied by the Penn World Tables 10.0.[12] The labor share is $1 - \alpha = 0.595$, the average for the period of interest in the same dataset. The capital share is $\alpha = 0.405$. Since $L = 1$, then $Z = (K)^{-\alpha} = 0.343$. The price indexation parameter ζ is set equal to 0.2, as in Lee (2020). The elasticity of substitution between intermediate goods is set equal to $\varepsilon_p = 9$ implying a steady-state mark-up of 12.5%. The price adjustment cost parameter is set equal to $\xi_p = 395.65$ as in Kyriazis (2022).

Capital Producers: Capital depreciates at rate $\delta = 0.015$. Steady state investment becomes $I = 0.2112$. The investment adjustment cost parameter is set equal to $\xi_I = 4$ as in Kyriazis (2022).

Monetary Authority: In the baseline scenario the economy is at the ZLB so $\phi_\Pi = \phi_Y = 0$ which then imply that $i_t^M = r = 0$. Outside of the ZLB, the inertia parameter in the Taylor rule is set equal to $\rho = 0.85$. The government assets held by the central bank at steady state are $qB^{CB}/4Y = 12.75\%$, in annual terms. The central bank transfer is determined at the steady state so that $T^{CB} = 0.0011$. The asset purchases coefficients are set equal to $\psi_\Pi = 11.48$ and $\psi_Y = 5$ as in Kyriazis (2022).

Fiscal Authority: Given that $r^B = 0.867\%$ per annum, the parameter that controls for the duration of long-term government bonds is set equal to $\gamma^* = 0.9771$ so as to match an average duration of 10 years / 40 quarters. The tax rate on labor is set equal to $\tau^L = 0.3$, while the tax rates on firms' dividends is set equal to $\tau^D = 0.35$. The ratio of government debt over GDP is set equal to $qB/4Y = 71.19\%$ which is the average value for the federal government debt held by the public during the period 2008-2021. The government spending-to-GDP ratio G/Y is 14.98% during the same period. The parameter controlling the adjustment speed of lump-sum taxes to public debt changes is set in the baseline scenario equal to $\phi^B = 0.001$ implying a weak reaction of taxes to government debt given that the economy is at the ZLB. Finally, the constant part of the lump-sum tax is determined at the steady state so as to satisfy the government budget constraint and is equal to $T = -0.029$.

[12] See Feenstra et al. (2015)

Table 4: Baseline Calibration of Parameter Values

Parameter	Description	Value	Target/Source
Households			
β^H	Foreign Household High Discount Factor	0.9903	$D = 2.401Y$
β^L	Foreign Household Low Discount Factor	0.9592	HtM Households Size = 8%
σ	Relative Risk Aversion Coefficient	2	Standard Value
ν	Inverse Frisch Elasticity	2	Standard Value
μ_L	Relative Weight of Labor Disutility	1.579	Internally Calibrated
χ_0	Portfolio Adjustment Cost Pivot	0.01	Baseline Scenario
χ_1	Portfolio Adjustment Cost Scale	45.336	$A = 13.349Y$
χ_2	Portfolio Adjustment Cost Curvature	1.998	WHtM Households Size = 14%
ρ_e	Autocorrelation of Earnings	0.966	McKay et al. (2016)
σ_e	St. Dev. of Log-Earnings	0.92	McKay et al. (2016)
$\bar{a}$	Upper Bound on a-grid	38.741	Top 10% Wealth Share = 71.3%
Labor Union			
ε_w	Elasticity of Substitution in Labor	9	Baseline Scenario
ξ_w	Wage Adjustment Cost Magnitude	1486.77	Kyriazis (2022)
Investment Bank			
λ_K	Divertible Fraction - Foreign Capital	0.345	Gertler and Karadi (2011)
λ_B	Divertible Fraction - Foreign Government Debt	0.0163	Gertler and Karadi (2013)
θ^{MF}	Bank Survival Rate	0.9966	Internally Calibrated
ω	Entering Banks Transfer Magnitude	0.001	Internally Calibrated
ξ_N	Management cost	0.0273	Internally Calibrated
Intermediate Goods Producers			
α	Capital Share of Income	0.405	Avg. Value 2008-2021
ε_p	Elasticity of Substitution Interm. Goods	9	Baseline Scenario
ξ_p	Price Adjustment Cost Magnitude	395.65	Kyriazis (2022)
Z	TFP	0.343	$Y = 1$
ζ	Price Indexation Degree	0.2	Lee (2020)
Capital Producers			
δ	Capital Depreciation Rate	0.015	Internally Calibrated
ξ_I	Investment Adjustment Cost Magnitude	4	Kyriazis (2022)
Monetary Authority			
ϕ_Π	Inflation Coefficient - Interest Rate Rule	0	ZLB
ϕ_Y	Output Coefficient - Interest Rate Rule	0	ZLB
ρ	Taylor Rule Inertia	0.85	Standard Value
ψ_Π	Inflation Coefficient - Asset Purchases Rule	11.48	Kyriazis (2022)
ψ_Y	Output Coefficient - Asset Purchases Rule	5	**?**
ρ^B	Asset Purchases Rule Inertia	0.85	Baseline Scenario
$qB^{CB}/4Y$	CB-Held Debt over GDP	12.75%	Avg. Value 2008-2021
Fiscal Authority			
γ	Duration Parameter	0.9771	Avg. Duration of 40 Quarters
ϕ_B	Debt Coefficient - Tax Rule	0.001	ZLB constraint
τ^L	Labor Income Tax Rate	30%	Standard Value
τ^D	Corporate Income Tax Rate	35%	National Accounts
G	Government Spending-to-Output Ratio	14.98%	Average Value 2008-2021
$qB/4Y$	Debt-to-GDP ratio	71.19%	Avg. Value 2008-2021

B.2 Computational Method

The model is solved using the sequence-space Jacobian methodology introduced by Auclert et al. (2021b). The logic behind this method is to break the model into different blocks containing the equations describing the behavior of each agent in the economy. Each block has the role of a function taking some aggregate sequences as inputs and producing some other aggregate sequnces as outputs.

In every period t, I select $[r_t, w_t, Y_t, \Pi_t, N_t, N_t^{MF}, B_t^{MF}, B_t^{CB}, K_t, K_t^{MF}, r_t^F, r_t^A]$. The 12 equations chosen as targets when solving for the 12 unknown variables are the Fisher equation (3.13), the WNKPC (3.9), the goods market clearing condition (3.61), the NKPC (3.44), the net worth evolution equation (3.17), the net worth evolution equation (3.31), the bond market clearing equation (3.60), the Taylor rule for asset purchases (3.52), the capital market clearing condition (3.59), the bank incentive constraint (3.30), the bank interest rate equation (3.33), and the equation for the illiquid return (3.21).